Assessment of Shear Strength of Geoploymer Concrete; A new approach to modify the Mohr-Coulomb failure envelopes

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ABSTRACT

The shear stress is evaluated as the functions of principal stresses produced under maximum stress conditions in the verge of the failure. Shear strength of Geopolymer concrete is evaluated by functions of failure envelopes under generic conditions may provide the higher value. The stress functions and conditions in Mohr-coulomb (MC) failure envelopes for isotropic materials are modified due to asymmetrical stress functions owing to anisotropic characteristic of the concrete. Several modifications have been incorporated over the basic failure envelopes depends on the practical significances and material properties. The analytical approach is introduced to evaluate the shear strength of Plain and Prestressed Geopolymer concrete by modified Mohr-Coulomb's (MMC) failure envelopes and substantiated with relevant code of practice for conventional concrete.

Keywords: Geopolymer concrete; Modified failure envelope; Mohr-Coulombs failure envelope; Prestressed concrete; Shear strength

1. Introduction

The Geopolymer concrete is a modern age building material being produced without using the conventional cement [1]. The mechanical properties of Geopolymer concrete is generally depends on the source materials, alkaline solution and aggregates used to prepare the mix [2]. There were immense experimental investigations have been reported on basic strength parameters of Geopolymer concrete but still there is a gap is encountered on the structural application due to lack

of specific code recommendations on the same. The shear strength is one of the important parameter of the materials used under structural applications. Some of the analytical tools have been formulated to assess the shear strength of the conventional concrete on the basic material properties such as compressive and tensile strength by experimental investigations. The preparation and the experimental investigations on basic mechanical properties of Geopolymer concrete discussed elsewhere [3].The test data of afore said investigations on basic mechanical properties are used to assess the shear strength of plane and prestressed concrete.

Mohr-Coulomb's failure envelopes provide reasonable prediction on shear strength of the concrete [4]. The Mohr's-Coulomb's envelopes could be drawn by connecting either linear or parabolic functions of stresses [5] for isotropic materials. Generalised Mohr-coulomb's failure envelope on biaxial state of stress representing the shear strength constituted by major and minor principal stresses ($\sigma_1 and \sigma_3$) without considering intermediate stresses ($\sigma_2 = 0$) [6]. The failure envelopes of general tri-axial test on concrete are elucidate with the conditions of $\sigma_2 = \sigma_3$ by hydrostatic pressure and the entire principal stresses being compressive [7]. The tri-axial stress analysis showing conservative or upper bound solution with experimental results for anisotropic or brittle materials especially concrete whose shear strength is depends on the nature and magnitude of the principal stresses ($\sigma_2 \neq \sigma_3 > 0$) over mutually perpendicular plane [8].In some of research works the failure envelops are construed by modifying basic Mohr-Coulomb's failure envelopes[9]. The modifications are essential when tensile stress partakes one of the parameter in the stress functions influencing the failure [10].

2. Criteria of failure

The failure criteria's are depends on the type of the materials and generally discussed under stress strain behaviours. If the material is elastic and ductile like steel and copper it shows linear deformation behaviour up to its yield point or under elastic limit and shows non-linear deformations till the material completely get strain hardening and reached to plastic state at which the material not resist further deformation and fails (Fig.2.1). If the material is elastic and brittle like cast iron and glass it may split or rupture before a distinct yield point reached. The materials may undergo sudden failure with intensive release of energy (Fig.2.2). The concrete neither fails like ductile material nor to do like plastic materials. Most of the experiment shows that concrete has discernible yield point before they fracture (Fig.2.3).



Fig 2.1Ductile failureFig 2.2 Plastic failureFig 2.3 Fracture failure

The criterion of the failure will be established by understanding the maximum serviceable strength of materials which are identified and utilized under specific design requirement. There may be several failure criteria's derived based on the assumptions and some of which are modified the conservative mode of failure. It's important to analyse material prior to enforce under design philosophy.

3. Analytical background on Mohr-Coulomb failure envelope

Mohr's theory (1900) on failure criteria does not show any differentiation on elastic deformation and brittle fracture. The failure of the material is represented by functional relationship with shear (τ) and normal (σ_n) stress acting on the plane, shown in fig 3.1 given the equation (Eq.3.1.1).

$$\tau = f(\sigma_n) \qquad \qquad \text{Eq. 3.1.1}$$

The functions are not revealed about the normal stresses stated as compressive or tensile in nature and also the magnitude of these stresses with their influencing factors towards failure under specific condition. The stress functions are generally inferred in a stress field represented in to three principal stresses, $\sigma_1 > \sigma_2 > \sigma_3$, the effect of σ_2 are neglected or made unique under the condition $\sigma_2 = \sigma_3$ to simplify stress functions into two dimensional (2D) attribute. When the samples of concrete subjected to bi-axial compressive stress, principal stresses (σ_1 and σ_3) are construed as Mohr's stresses of circle and Mohr's failure envelope is drawn as shown in fig 3.2.



Fig 3.1 Functional relationship by Mohr circles of stresses



Fig 3.2 Failure envelope on series of Mohr's circle of stresses

The relation of normal and shear stresses of Mohr's failure envelopes are expressed as nonlinear function given in Eq.3.1.(2&3)

Coulomb's (1776) proposed the linear functional relationship of shear strength of soil [8] as

Coulomb considered that the shear strength and normal stress could be represented by straight line. The Mohr's theory also state that the shear strength is depends on the normal stress but the functional relationship is not linear. The MC criterion can be considered as a contribution from Mohr and Coulomb [5] Combining the Mohr failure criterion with the Coulomb equation gave a straight line tangent [12], shown in fig 3.3



Fig 3.3 Mohr-Coulomb failure line

When the principal stresses are compressive (C₀), the failure plane can be projected by functions of MC failure envelope on available experimental data. The material strength is varies with nature of stress on uniaxial and bi axial planes. Concrete possess higher strength against failure under uniaxial as well as bi axial compression [13]. The tensile strength of the brittle material is effectively partake the failure conditions. If one of the principal stress is tensile (T₀) in nature and having the ratio of $C_0/T_0 > 10$; some modification is needed to project the theoretical tensile strength of the material [10].

4. Modifications of (MC) failure envelopes

The tensile strength of the material is projected by modifying the principal stresses shown in fig 4.1. MC failure envelope drawn over the Mohr's circle of stresses C and D. The modified Mohr's circle of stress T0 and C0 is [formulated] drawn under the relative function of same angle of failure plane (α_f), angle of tangent (ϕ) and formulations on modifications are shown as below



Fig. 4.1 Modified Mohr- Coulomb (MC) failure envelope

$$\boldsymbol{\sigma} = \frac{\sigma_1 + \sigma_3}{2}, \, \boldsymbol{\tau} = \frac{\sigma_1 - \sigma_3}{2} \text{ Considering the maximum angle of failure } (\alpha_f = 45^\circ)$$
$$\boldsymbol{m} = \frac{C_0}{T_0} = \frac{(1 + \sin\phi)}{(1 - \sin\phi)}; \quad C_0 = \frac{m}{m+1}; \quad T_0 = \frac{C_0}{2}(1 - \sin\phi);$$

The MC failure envelope is modified by using parabolic intrinsic curve [14] to reflect the convexity of the failure of the envelope, shown in fig 4.2. The relations of shear and normal stresses of the concrete shown[14,15] as



Fig. 4.2 Intrinsic parabolic failure envelope

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$$\frac{\sigma_n}{f_t'} + \frac{1}{K} \left(\frac{\tau_{ci}}{f_{c'}}\right)^2 = 1$$
 Eq 4.1

Where f'_c and f'_t are effective compressive strength and tensile strength of concrete respectively

$$K = \frac{1}{4} \left(m + 2 \left(1 - \sqrt{m+1} \right) \right)$$
 Eq 4.2

$$m = \frac{f_{c'}}{f_{t'}} = \frac{v_c f_c}{v_t f_t}$$
 Eq.4.3

Where f_c and f_t are compressive strength and tensile strength of concrete, v_c and v_t effectiveness factors respectively. The value 'K' determined by intrinsic parabolic failure curve tangent to the Mohr's circle of compression and tension. The value of normal stresses and shear stresses from intrinsic functions of parabola is given as

$$\sigma_n = (1 - \cot^2_{\alpha})f_t$$
 Eq.4.4

$$\tau_{\rm ci} = \frac{2{\rm K} {\rm f}_{\rm t'}}{{\rm tan} \alpha_f} \qquad \qquad {\rm Eq.4.5}$$

This equation gives upper bound solution for shear strength when $\alpha_f < 45^\circ$ and the corresponding equation of principal stresses give as

$$\sigma_1 = (1 - K(cosec\alpha_f - 1)^2 f_t)$$
 Eq.4.6

$$\sigma_3 = (1 - K(cosec\alpha_f + 1)^2 f_t)$$
 Eq.4.7

The theoretical angle of failure plane, α_f are ranging from 37° to 90° (37° < α_f < 90°, for $\sigma_1 = f_c$ and $\sigma_3 = f_t$). The value of shear strength is the function of $sin2\alpha_f$ as given in equation, Eq.3.1.3. hence the maximum α_f considered as 75° in line with experimental investigations. Then the shear strength of concrete under maximum probable shear failure plane α_f as

$$\tau_{\rm ci} = 0.53 \text{K} f_t$$
 Eq.4.8

$$\tau_{\rm ci} = 0.53 f_t \quad ({\rm K} \cong 1) \qquad \qquad {\rm Eq.4.8}$$

5. Formulation of shear strength of concrete on 2D state of stresses

The shear stress of the concrete is assessed by modifying Mohr's failure criteria angle of tangent of failure envelopes considering the principal stresses σ_1 and σ_2 is drawn by Mohr's circles of stresses for compressive strength and tensile strength of concrete respectively. Here σ_2 is representing maximum principal tensile strength to avoiding repetition of symbols σ_1 and $\sigma_3 = 0$ since the circle passes through origin. The Principal compressive stress is drawn in a positive quadrant and

Principal tensile stress is in negative quadrant of the axis. The Mohr's coulomb failure envelope is drawn as the tangent of compressive and tensile stress circles [16] shown in fig 4.3. The value of shear stress is the function of principal stresses measured between the points of intersection of the tangent of the circles on y-axis to the origin.

When $\mathbf{\tau}_{c} = \mathbf{f}(\sigma_{1}, \sigma_{2})$ Graphically $\tau_{c} = ED$ $\tau_{c} = \frac{\sigma_{1}}{2}$; When $\sigma_{2} = 0$ $\tau_{c} \neq 0$; Even if $\sigma_{2} = 0$

The value of τ_c is existed even the principal tensile stress is zero. The concrete is generally weak in tension and can be considered as critical strength parameter rather than compressive stress parameter. The concrete structure will fail even though it has good compressive strength when the principal tensile stress is reached to its maximum limit. Under this condition, the shear strength of the concrete could be considered on the tensile strength of the concrete. A new approach is introduced here over modified failure envelope for plain concrete (MFE-PC)[17] by drawing the triangle AO₁O₂ with angle Φ as shown in fig.5.1.



Fig 5.1-Modification of tangent angle of failure envelope-2D

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In this approach, the shear strength of the material is zero when the tensile strength of material tends to zero.

The shear strength from the modified approach can be assessed as explained below.

$$\tau_{\rm c} = \tau_{\rm cf}$$
 is the modified function (f_m) of σ_1, σ_2 and ϕ
 $\sigma_1 = f_c; \sigma_2 = f_t$
 $\tau_{cf} = f_m (\sigma_1, \sigma_2, \phi)$

Graphically $\tau_{cf} = CD$

$$tan\Theta_1 = \frac{R_2}{2(\sqrt{R_1 R_2})}$$
 Eq.5.1

$$tan \Theta_2 = \frac{R_2 - R_1}{2(\sqrt{R_1 R_2})}$$
 Eq.5.2

$$R_2 = \frac{\sigma_1}{2}$$
 Eq.5.3

$$R_1 = \frac{\sigma_2}{2}$$
 Eq.5.4

$$\Phi = \Theta_1 - \Theta_2$$
 Eq.5.5

If $R_1 \rightarrow 0$; $tan\Phi = 0$, then $\tau_{cf} = 0$ When $\sigma_2 = 0$ then $\phi = 0$ and $\tau_{cf} = 0$; this condition is mathematically modelled to show that the material if fails due to tension ,the shear strength of material become zero and no longer to take further shear stresses even though the material is not reached to maximum compressive strength.



6. Formulation of shear strength of concrete 3D state of stresses

Fig. 6.1-Modification of angle of tangent of failure envelope-3D

The modified failure envelope for prestressed (MFE-PSC) concrete is formulated by considering f_c and f_p be the bi axial compressive stress and f_t is considered as tensile strength of the concrete

$$\sigma_z = f_p \ (\sigma_1 > \sigma_3 > 0), \sigma_x = f_c \ (\sigma_1 > \sigma_3 = 0), \sigma_y = f_t (\sigma_2 > \sigma_3 = 0); \ \tau_c = \tau_{cf-3D}$$

$$\tan \Theta_1 = \frac{R_2}{2(\sqrt{R_1 R_2})}$$
 Eq.6.1

$$tan\Theta_2 = \frac{R_2 - R_1}{2(\sqrt{R_1 R_2})}$$
 Eq.6.2

$$tan \Theta_3 = \frac{R_2 - R_3}{2(\sqrt{R_2 R_1})}$$
 Eq.6.3

$$R_2 = \frac{\sigma_x}{2}$$
 Eq.6.3

$$R_1 = \frac{\sigma_y}{2}$$
 Eq.6.4

$$R_3 = \frac{\sigma_z}{2}$$
 Eq.6.5

$$\phi = (\theta_1 - \theta_2)$$
 Eq.6.6

$$CD = \frac{\sigma_1}{2} tan\phi$$
 Eq.6.7

$$GC \cong CD$$
 Eq.6.8

$$\Psi = (tan\theta_2 - tan\theta_3)$$
 Eq.6.9

$$HG = \frac{\sigma_2}{2} (tan\psi)$$
 Eq.6.10

$$\tau_{cf-3D} = CD + GC + HG$$
 Eq.6.11

Let $R_3 = R_1$, GC = 0 for 2D. $\psi = 0$; when $R_3 = 0$ then $\tau_{cf-3D} = CD = \frac{\sigma_1}{2} \tan \Phi$ (reducing to 2D conditions, Eq.5.6)

7. Corroboration of formulation

The shear strength of concrete as a function of compressive strength and tensile strength in 2D is determined under the condition that the concrete would fail when tensile stress exceeds the principal tensile stress. The tensile strength and compressive strength of concrete arrived by experimental investigations of split tensile strength of concrete and compressive strength test conforming to relevant code of practice [18]. The direct tensile strength of the concrete can be considered 90% of split tensile strength [19].

The shear strength under the functional relationship $\tau_c = f_m (\sigma_1, \sigma_2, \phi)$, Eq.5.6 is compared with the equation suggest by ACI-318 [20], Eq. 7.1, shown in table 7.1 and fig 7.1.

Mix	f _c MPa	f _t MPa	τ _{ci} (IPF) MPa	τ _{cf} (MFE-PC) MPa	τ _c (ACI-318) MP a	$\frac{\text{Variations}}{\frac{(\tau_{cf} - \tau_c)}{\tau_c} \times 100}$ (%)
FGC-M1	13.35	1.88	0.79	0.49	0.54	-10.20
FGC-M2	15.69	2.13	0.90	0.55	0.59	-727
FGC-M3	18.56	2.44	1.03	0.63	0.64	-1.58
FGC-M4	22.75	3.08	1.30	0.80	0.71	11.25
FGC-M5	26.35	3.52	1.49	0.90	0.76	15.5
FGC-M6	28.56	3.67	1.55	0.94	0.79	15.95
FGC-M7	31.98	4.12	1.74	1.05	0.84	20
FGC-M8	33.69	4.33	1.83	1.11	0.86	22.52
FGC-M9	35.58	4.53	1.92	1.15	0.88	23.47
FGC-M10	37.64	4.65	1.97	1.18	0.91	22.88
FGC-M11	40.23	5.14	2.17	1.32	0.94	28.78
FGC-M12	41.54	5.33	2.25	1.37	0.96	29.92

Table 7.1 Shear strength of plain Geopolymer concrete – (2D state of stresses)



Fig. 7.1 Shear strength of Geopolymer concrete for plain concrete

Suggested by ACI 318-05 for conventional concrete, where f_{cy}' is the cylinder compressive strength equivalent to 80% of cube compressive strength of concrete (f_{cy}) . The shear strength of concrete under external compression or under prestress depends on the compressive strength of concrete at the time of transfer of stresses on concrete. The shear strength of concrete for prestressed concrete especially for beam case suggested under assumption that the shear crack or shear deformation might be occurred when the principal stress reaches the approximate value of $0.3\sqrt{f_{cy}}$ [21] with the multiplication factors ' λ ' [22] based on the type of the concrete.

The shear strength of the concrete in the beam under prestressing can be considered as

$$\boldsymbol{\tau_{cp1}} = [0.3\lambda\sqrt{f_{cy}} + 0.3f_{cy}]$$
 Eq.7.2

The British code and Indian code [23] provision on the shear strength of the concrete under the same context of prestressing is given as

$$\boldsymbol{\tau_{cp2}} = [0.67 \times \sqrt{(f_t^2 + 0.8f_{cp}f_t)}]$$
 Eq.7.2

The shear strength from modified failure envelope are compared with BSEN:1992 and ACI-318M shown in the table 7.2 and fig 7.2.

Mix	f _c MPa	f _t MPa	f _{cp} MPa	<i>f_{cp1}</i> (BSEN:1992) MPa	<i>f_{cp2}</i> (ACI-318) MPa	$ au_{cf-3D}$ (MFE-PSC) MPa
FGC-M1	13.35	1.88	5.16	2.53	2.25	1.15
FGC-M2	15.69	2.13	5.16	2.61	1.97	1.24
FGC-M3	18.56	2.44	5.16	2.70	2.13	1.36
FGC-M4	22.75	3.08	5.16	2.83	2.47	1.60
FGC-M5	26.35	3.52	5.16	2.93	2.72	1.87
FGC-M6	28.56	3.67	5.16	2.98	2.81	1.94
FGC-M7	31.98	4.12	5.16	3.07	3.08	2.13
FGC-M8	33.69	4.33	5.16	3.11	3.20	2.25
FGC-M9	35.58	4.53	5.16	3.15	3.33	2.37
FGC-M10	37.64	4.65	5.16	3.19	3.40	2.46
FGC-M11	40.23	5.14	5.16	3.25	3.70	2.50
FGC-M12	41.54	5.33	5.16	3.28	3.82	2.74

 Table 7.2 Shear strength of prestressed Geopolymer concrete (3D state of stresses)



Fig. 7.2 Shear strength of prestressed Geopolymer concrete

8. Conclusion

1. The shear strength of concrete evaluated as functions of compressive strength provides the lower bound solution for higher grade concrete.

2. The practical significance of the shear stress is based on the prevailing tensile stresses developed in the concrete due to the applications of load and also depends on the type of structural failure criteria. The shear strength of the concrete is formulated based on principal tensile stresses

3. The functional shear strength of concrete increases as the bi axial compressive stress increases maximum limit of compressive stress in concrete.

4. Shear strength by Intrinsic parabolic solutions provide upper bound solution or highly conservative value compared to the respective ACI-318 equation and modified approach discussed in this paper. The shear strength of concrete found less than half of the tensile strength of concrete shown through ACI-318 equations and modified approach

5. The shear stress determined by modified failure envelope (MFE-PC) were observed +15% to - 10%, 15% to 20%, and 20% to 30% variations with ACI suggested equations for concrete strength ranging from M15 to M25, M25 to M30 and M30 to M40 grade respectively.

6. The shear strength of the concrete mix grade higher than M-25 presented comparatively higher shear strength by modified failure envelopes. The shear strength of concrete would be significantly more for higher grade concrete. The general equation on shear strength wouldn't be justified under the remarks of the results for high-grade concrete unless modifying the equation or experimental formulation on the range of strength of concrete.

7. The modified failure envelopes are substantiating the shear strength of concrete which increases due to prestressing effect.

8. The value of shear strength from BSEN and ACI-318 codes varies with -10% to 30 %. The shear strength of prestressed concrete by modified failure envelopes(MFE-PSC) provides considerably 30 to 50% lower value of strength. The value can be reliable since lower bound solution compare to value from BSEN and ACI-318 codes.

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The authors declare that they have no conflict of interest

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