Thermal Impermanence of a couple stress visco-elastic Walter’s (modal B’) nanofluid layer through a porous medium

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Abstract
Thermal Impermanence of a couple stress visco-elastic Walter’s (modal B’) nanofluid layer through a porous medium is investigated for more realistic boundary conditions. By applying Perturbation method, Normal mode technique, the effects of the various physical parameters of the system namely Lewis number, modified diffusivity ratio, nano particle Rayleigh number and couple stress on the stationary convection have been investigated both analytically and graphically. The Lewis number, modified diffusivity ratio and nano particle Rayleigh number are found to have stabilizing effect, whereas couple stress has a destabilizing effect for stationary convection.

Keywords
Nanofluid; Walter’s (modal B’) visco-elastic fluid; Normal mode technique; Rayleigh number; Lewis number; modified diffusivity ratio; couple stress.

1. Introduction
Chandrasekhar [1] has explained in detail the thermal impermanence of a Newtonian fluid under the assumptions of hydrodynamics and hydromagnetics. There are many visco-elastic fluids that cannot obey Maxwell’s constitutive relations. One such type of fluid is Walter’s (modal B’) fluid. D.Kapil and S. Kumar [2] discussed hydromagnetic instability of visco-elastic Walter’s (modal B’) nanofluid layer heated from below and found that magnetic field has a stabilizing effect for stationary convection. The effect of compressibility and suspended particles on thermal convection in a Walter’s (modal B’) visco-elastic fluid in hydromagnetics has been studied by Sharma and Sharma [3]. In recent years, the study of flow of fluids through porous medium pay an important role due the recovery of crude oil from the pores of reservoir rocks. The flow through porous medium having interest for petroleum engineers and geophysical fluid dynamists. Sharma and Sunil [4] have investigated the thermal impermanence of an Oldroydian visco-elastic fluid with suspended particles in hydromagnetics in a porous medium. Rayleigh Bénard deformation in visco-elastic Rivlin-Ericksen nanofluid in the presence of suspended particles has been studied by S. Kumar and D.Kapil [5].S.K. Pandir et al. [6] have discussed the effect of rotation on hydromagnetic instability of visco-elastic Walter’s (modal B’) nanofluid layer heated from below and found that there exist destabilizing effect of rotation on stationary convection. A solid with holes and characterized by the manner in which the holes are imbedded, is known as porous medium. The flow through the porous medium is governed by Darcy’s law which states that the usual viscous term in the equation of motion of Walter’s (modal B’) nanofluid is replaced by resistance term

\[-\frac{1}{k_1}(\mu - \mu' \frac{\partial}{\partial t})\nabla^2 q_f\text{, where }\mu \text{ and } \mu' \text{ are the viscosity and visco-elasticity of incompressible Walter’s (model B’) fluid, }k_1 \text{ is the medium permeability and } q \text{ is the Darcian velocity of the fluid.}\]

Couple-stress theory has discussed by Stokes [7]. Couple-stress theory having a great importance in various industrial fluids, medical field such as lubrication mechanism, functioning of synovial joints during human locomotion and opened new ways in several fields of scientific and technical research. Sharma and Sharma [8] have studied the effect of suspended particles on couple-stress fluid heated from below in the presence of vertical rotation and vertical magnetic field and found that the effect of rotation is to stabilize the system, whereas suspended particles have destabilizing effects. Thermal instability problems in a porous layer saturated by a nanofluid employing the Brinkman-modal for the porous medium has been studied by Kuznetsov and Nield [9]. Shiva Kumar et
al.[10] studied the effect of rotation on thermal deportation in a couple-stress fluid saturated rotating rigid porous layer. Keeping in the mind the importance of couple-stress fluid, the present paper attempts to studied thermal impermanence of a couple-stress visco-elastic Walter’s (modal B’) nanofluid layer through porous medium.

2. Mathematical Formulation

Suppose an infinite horizontal layer of couple-stress visco-elastic Walter’s (modal B’) nanofluid of thickness \( d^* \) is bounded by \( z = 0 \) and \( z = d^* \) and heated from below. The fluid layer is acting in upward direction under gravity force \( g \) \((0, 0, -g)\). \( T_0 \) and \( \varphi_0 \) are the temperature and volumetric fraction of nano particles at \( z = 0 \) and \( T_1, \varphi_1 \) are temperature and volumetric fraction at \( z = d^* \) respectively.

The governing equation for couple-stress visco-elastic Walter’s (modal B’) nanofluid

\[
\nabla q_f = 0
\]

\[
\frac{\rho}{\epsilon} \frac{d q_f}{d t} = -\nabla p + \rho g - \frac{1}{k_1} \left( \mu - \mu' \frac{\partial}{\partial t} \right) \nabla^2 q_f - \frac{1}{k_1} (\delta - \delta' \nabla^2) q_f
\]

where \( \frac{d}{dt} = \frac{\partial}{\partial t} + (q_f \cdot \nabla) \) stands for convection derivative, \( q_f \) is the velocity vector, \( p \) is the hydrostatic pressure, \( \mu \) and \( \mu' \) are the viscosity and visco-elasticity \( \delta \) and \( \delta' \) are fluid viscosity and couple-stress fluid viscosity and \( g(0, 0, -g) \) is acceleration due to gravity. The density \( \rho \) of nanofluid can be written as

\[
\rho = \varphi \rho_p + (1 - \varphi) \rho_f
\]

where \( \varphi \) is the volume fraction of nano particles, \( \rho_p \) and \( \rho_f \) are the densities of nano particles and base fluid respectively.

The equation of motion for couple-stress visco-elastic Walter’s (modal B’) nanofluid is given as:

\[
\frac{\rho}{\epsilon} \frac{d q_f}{d t} = -\nabla p + \left( \varphi \rho_p + (1 - \varphi) \rho (1 - \alpha (T - T_0)) \right) g - \frac{1}{k_1} \left( \mu - \mu' \frac{\partial}{\partial t} \right) \nabla^2 q_f - \frac{1}{k_1} (\delta - \delta' \nabla^2) q_f
\]

where \( \alpha \) is the coefficient of thermal expansion.

The continuity equation for the nano particles is

\[
\frac{\partial \varphi}{\partial t} + \frac{1}{\epsilon} q_f \nabla \varphi = D_B \nabla^2 \varphi + \frac{D_T}{\tau_i} \nabla^2 T
\]

where \( D_B \) is the Brownian diffusion coefficient and \( D_T \) is the Thermoporetic diffusion coefficient of the nano particles.

The energy equation in nanofluid is

\[
\rho_c \left( \frac{\partial T}{\partial t} + q_f \nabla T \right) = k_m \nabla^2 T + \epsilon (\rho_c)_p \left( D_B \nabla \varphi \cdot \nabla T + \frac{D_T}{\tau_i} \nabla T \cdot \nabla T \right)
\]

Where \( \rho_c \) is the heat capacity of fluid, \( (\rho_c)_p \) is the heat capacity of nano particles and \( k_m \) is the thermal conductivity.

Introducing non-dimensional variables as:
\( (x', y', z') = \left( \frac{x, y, z}{d^*} \right), \)

\[
q_f'(u', v', w') = q_f \left( \frac{u, v, w}{k} \right) d^*, \quad t' = \frac{tk}{d^{*2}},
\]

\[
p' = \frac{p}{\rho k^2} d^{*2}, \quad q' = \frac{q - q_0}{\varphi_1 - \varphi_0},
\]

\[
T' = \frac{T - T_0}{T_0 - T_1},
\]

where \( \frac{k_m}{\rho_c} = k \) is the thermal diffusivity of the fluid.

Equations (1), (4), (5) and (6), in non-dimensional form can be written as:

\[
\nabla q_f = 0 \quad (7)
\]

\[
\frac{1}{\rho \varepsilon} \frac{\partial q_f}{\partial t} = -\nabla p - R_a m \hat{e}_z - R_a n q \hat{e}_z - R_a T \hat{e}_z - \frac{1}{k_1} (1 - nF) \nabla^2 q_f - \frac{1}{\rho l} (1 - y \nabla^2) q_f \quad (8)
\]

\[
\frac{\partial \varphi}{\partial t} + \frac{1}{\varepsilon} q_f \nabla \varphi = \frac{1}{k_1} \nabla^2 \varphi + \frac{N_A}{k_1} \nabla^2 T \quad (9)
\]

\[
\frac{\partial T}{\partial t} + q_f \nabla T = \nabla^2 T + \frac{N_B}{k_1} \nabla \varphi \cdot \nabla T + \frac{N_A N_B}{k_1} \nabla \varphi \cdot \nabla T \quad (10)
\]

[The dashes (') have been dropped for simplicity]

Here non-dimensional parameters are:

- Lewis number \( L_e = \frac{k}{d_B}, \) Prandtl number \( \rho r = \frac{\mu}{\rho k} \), Thermal Rayleigh number \( R_a = \frac{\rho g a d^3}{\mu k} (T_0 - T_1), \)
- Basic- density Rayleigh number \( R_{\rho m} = \frac{[\rho \varphi_0 + \rho (1 - \varphi_0)] d^3}{\mu k}, \) Nano particle Rayleigh number \( R_{\rho n} = \frac{[\rho_p - \rho] (\varphi_1 - \varphi_0) d^3}{\mu k}, \) Kinematic visco-elasticity parameter \( F = \frac{\mu'}{\rho d^2}, \) Modified diffusivity ratio \( N_A = \frac{d_T}{d_B (\varphi_1 - \varphi_0)} (T_0 - T_1), \) Modified particle density increment \( N_B = \frac{\varepsilon (\rho_c) p (\varphi_1 - \varphi_0)}{(\rho_c) f}, \) Modified couple-stress parameter \( = \frac{\mu'}{\mu d^2} \), dimensionless medium permeability \( \rho l = \frac{k}{d^2}. \)

We assume that temperature and volumetric fraction of nano particles are constant on boundaries. Thus the dimensionless boundaries conditions are

\[
w = 0, \quad T = 1, \quad \varphi = 0 \quad at \quad z = 0
\]

and \( w = 0, \quad T = 0, \quad \varphi = 1 \quad at \quad z = 1 \quad (11)\)

### 2.1) Basic States and its solution

The basic state of nanofluid is supposed to be time independent of time and can be written as \( q_f'(u, v, w) = 0, \) \( p' = p(z), \) \( T' = T_b(z), \) \( \varphi' = \varphi_b(z), \) Equations (7) to (10) using boundary conditions (11) give solution as:

\[
T_b = 1 - z \quad and \quad \varphi_b = z \quad (12)
\]
2.2) Perturbation solution

The stability of the system can be studied by introducing small perturbations to primary flow, and written as

\[ q_f'(u, v, w) = 0 + q_f''(u, v, w), \quad T' = T_b + T'', \quad \varphi' = \varphi_b + \varphi'', \quad p' = p_b + p'', \quad \text{with} \ T_b = 1 - z \quad \text{and} \quad \varphi_b = z \]  \hspace{1cm} (13)

Using equation (13) in equation (7) to (10) and linearize by neglecting the product of the prime quantities, we obtain the following equations:

\[ \nabla q_f = 0 \]  \hspace{1cm} (14)

\[ \frac{1}{p_{r'e}} \frac{\partial w}{\partial t} \tilde{e}_z = - \nabla p - R_{am} \tilde{e}_z R_{an} \varphi \tilde{e}_z - R_a T \tilde{e}_z - \frac{1}{k_1} (1 - nF) \nabla^2 w \tilde{e}_z - \frac{1}{p_l} (1 - \gamma \nabla^2) w \tilde{e}_z \]  \hspace{1cm} (15)

\[ \frac{\partial \varphi}{\partial \varepsilon} + \frac{1}{\varepsilon} \frac{\partial \varphi}{\partial t} = \frac{1}{l_e} \nabla^2 \varphi + \frac{N_B}{l_e} \nabla^2 T \]  \hspace{1cm} (16)

\[ \frac{\partial T}{\partial \varepsilon} - \frac{\partial \varphi}{\partial t} = \nabla^2 T + \frac{N_B}{l_e} \left( \frac{\partial T}{\partial z} - \frac{\partial \varphi}{\partial z} \right) - 2 \frac{N_A N_B}{l_e} \frac{\partial T}{\partial z} \]  \hspace{1cm} (17)

The dashes (‘’) have been dropped for simplicity.

Since \( R_{am} \) is just a measure of basic static pressure gradient so it is not involved in these and subsequent equations. Now by operating Eq. (15) with \( \tilde{e}_z \cdot \text{curl curl}, \) we get:

\[ \left[ \frac{1}{k_1} (1 - nF) - \frac{\gamma}{p_l} \right] \nabla^4 w + \frac{n}{p_{r'e}} \nabla^2 w + \frac{1}{p_l} w = R_a \nabla^2 H - R_{an} \nabla^2 H \varphi \]  \hspace{1cm} (18)

where \( \nabla^2_H = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \) is the two dimensional Laplacian operator on horizontal plane.

3. Normal mode observation

For observing the disturbances in to normal modes and assuming that the perturbed quantities are of the form:

\[ [W, T, \varphi] = [W(z), T(z), \varphi(z)] \exp(ik_x x + ik_y y + nt) \]  \hspace{1cm} (19)

Where \( k_x \) and \( k_y \) are wave numbers in \( x \) and \( y \) directions respectively, while \( n \) is growth rate of disturbances.

Using eq. (19), eq. (16), (17) and (18) become:

\[ \frac{1}{\varepsilon} W - \frac{N_A}{l_e} (D^2 - a^2) T = \left[ \frac{1}{l_e} (D^2 - a^2) - n \right] \varphi = 0 \]  \hspace{1cm} (20)

\[ W + \left[ (D^2 - a^2) - n + \frac{N_B}{l_e} D - \frac{2N_A N_B}{l_e} D \right] T - \frac{N_B}{l_e} D \varphi = 0 \]  \hspace{1cm} (21)

\[ \left[ \frac{1}{k_1} (1 - nF) - \frac{\gamma}{p_l} \right] (D^2 - a^2)^2 + \frac{n}{p_{r'e}} (D^2 - a^2)^2 + \frac{1}{p_l} W + a^2 R_a T - a^2 R_{an} \varphi = 0 \]  \hspace{1cm} (22)

Where \( D = \frac{d}{dz} \) and \( a = \sqrt{k_x^2 + k_y^2} \) is the dimensionless the resultant wave number. The boundary conditions of the problem in view of normal mode are written as

\[ W = 0, D^2 W = 0, \quad T = 0, \quad \varphi = 0 \text{ at } z = 0 \text{ and } W = 0, D^2 W = 0, \quad T = 0, \quad \varphi = 0 \text{ at } z = 1 \]  \hspace{1cm} (23)
4. Linear Stability Observation

Consider the solution in the form \( w, T, \varphi \) is given as:

\[
 w = w_0 \sin \pi z, T = T_0 \sin \pi z, \varphi = \varphi_0 \sin \pi z
\]

Equations (20), (21) and (22) reduced as

\[
\frac{1}{\varepsilon}w_0 + \frac{Na}{\varepsilon}JT_0 + \left[ \frac{1}{L_e}J + n \right] \varphi_0 = 0
\]

\[
 w_0 - (J + n)T_0 = 0
\]

\[
 \left[ \left( \frac{1}{k_1} (1 - nF) - \frac{\gamma}{p_f} \right) J^2 - \frac{n}{p_r \varepsilon} J + \frac{1}{p_f} \right] w_0 - a^2 R_a T - a^2 R_{an} \varphi = 0
\]

From equation (24) & (25), we get

\[
 \left[ \frac{1}{\varepsilon} (J + n) + \frac{Na}{\varepsilon} J \right] T_0 + \left( \frac{1}{L_e} J + n \right) \varphi_0 = 0
\]

From equation (25), (26) & (27), we get

\[
 R_a = \frac{1}{\varepsilon^2} \left[ \left( \frac{1}{k_1} (1 - nF) - \frac{\gamma}{p_f} \right) J^2 - \frac{n}{p_r \varepsilon} J + \frac{1}{p_f} \right] \left( \frac{1}{L_e} \varphi + \frac{Na}{L_e} \right) R_{an}
\]

where \( J = \pi^2 + 2 \)

For neutral stability, the real part of \( n \) is zero. Hence, on putting \( n = i \omega \), \( \omega \) is the real and dimensionless frequency of oscillation) in eq.(28), we get:

\[
 R_a = \Delta_1 + i \omega \Delta_2
\]

where

\[
 \Delta_1 = \frac{J}{\varepsilon^2} \left[ \left( \frac{1}{p_f} + \omega^2 \right) + \left( \frac{\gamma}{p_r \varepsilon} + \frac{\omega^2 F}{k_1} \right) \right] + \frac{1}{\left( \frac{L_e}{\varepsilon^2} \right) + \omega^2} \left[ \frac{J^2 (1 + \frac{Na}{L_e})}{L_e} \varphi + \frac{1}{\varepsilon} \omega^2 \right] R_{an}
\]

and imaginary part

\[
 \Delta_2 = \frac{1}{\varepsilon^2} \left[ \left( \frac{1}{k_1} - \frac{\gamma}{p_f} - \frac{1}{p_r \varepsilon} \right) J^2 - \frac{FJ^3}{k_1} + \frac{1}{p_f} \right] + \frac{J \left( \frac{1}{L_e} \right) - \frac{\omega^2}{\varepsilon} \left( \frac{Na}{L_e} \right)}{\left( \frac{L_e}{\varepsilon^2} \right) + \omega^2} R_{an}
\]

\( R_a \) will be real since it is a physical quantity. Hence, it follow from Eq.(36) that either \( \omega = 0 \) (exchange of stability, steady state) or \( \Delta_2 = 0 \) (\( \omega \neq 0 \) overstability or oscillatory onset).

5. Stationary Deportation

When the stability occurs in as stationary convection, the marginal state will be characterized by \( \omega = 0 \). the Eq.(29) reduces as:

\[
 (R_a)_{st} = \frac{\left( \pi^2 + a^2 \right)}{\varepsilon^2} \left[ \left( \frac{1}{k_1} - \frac{\gamma}{p_f} \right) \left( \pi^2 + a^2 \right) + \frac{1}{p_f} \right] + \left( \frac{L_e}{\varepsilon} + Na \right) R_{an}
\]

Here \( R_a \) is independent of both the prandtl numbers and the parameters containing the Brownian effects and the thermophoretic effects and presented in the thermal energy equation and the conversation equation for nano particles.
Take $x = \frac{a^2}{\pi^2}$ in Eq. (32), then we have

$$
(R_A)_n = \frac{(1 + x)}{x} \left[ \left( \frac{1}{k_1} - \frac{\varphi}{p_1} \right) (1 + x)^2 \pi^4 + \frac{1}{p_1} \right] + \left( \frac{L_e}{\varepsilon} + N_A \right) R_n 
$$

(33)

To study the effects of Lewis number $L_e$, modified diffusivity ratio $N_A$, and nano particles Rayleigh number $R_n$ and couple-stress on stationary convection. We examine the nature of

$$
\frac{\partial R_A}{\partial L_e}, \frac{\partial R_A}{\partial N_A}, \frac{\partial R_A}{\partial \sigma_n}, \frac{\partial R_A}{\partial \gamma}
$$

analytically.

From eq. (33)

$$
\frac{\partial R_A}{\partial L_e} > 0, \frac{\partial R_A}{\partial N_A} > 0, \frac{\partial R_A}{\partial \sigma_n} > 0, \frac{\partial R_A}{\partial \gamma} < 0
$$

It implies that for stationary convection Lewis number, modified diffusivity ratio and nano particle Rayleigh number have stabilizing effect whenever couple-stress has destabilizing effect on the fluid layer.

### 6. Results and discussion

The effect of couple-stress on thermal Impermanence of visco-elastic Walter’s (modal B’) nanofluid layer heated from below is investigated under realistic boundary conditions.

Figure 1 represents the variation of stationary Rayleigh number with Lewis number $L_e$ for different values of $p_1$. The stationary Rayleigh number $R_n$ is plotted against Lewis number for fixed values of $N_A = 5, \varepsilon = .1, L_e = 30, \gamma = 5$ and different values of $R_{\sigma_n} = 5, 10, 15, k_1 = 1, 2, 3$ The Rayleigh number increases with increases in Lewis number, which shows that Lewis number has stabilizing effect on the stationary deformation.

Figure 2 represents the variation of stationary Rayleigh number with Lewis number $L_e$ for different values of $L_e$. The stationary Rayleigh number $R_n$ is plotted against Lewis number for fixed values of $N_A = 5, \varepsilon = .1, k_1 = 1, \gamma = 5$ and different values of $R_{\sigma_n} = 5, 10, 15, L_e = 1, 2, 3$ The Rayleigh number increases with increases in Lewis number, which shows that Lewis number has stabilizing effect on the stationary deformation.

Figure 3 represents the variation of stationary Rayleigh number with modified diffusivity number $N_A$ for different values of $L_e$. The stationary Rayleigh number $R_n$ is plotted against $N_A$ for fixed values of $N_A = 5, \varepsilon = .1, k_1 = 1, \gamma = 5$ and different values of $R_{\sigma_n} = 50, 100, 150, L_e = 1, 2, 3$ The Rayleigh number increases with increases in $N_A$, which shows modified diffusivity number $N_A$ has stabilizing effect on the stationary deformation.

Figure 4 represents the variation of stationary Rayleigh number with modified diffusivity number $N_A$ for different values of $\gamma$. The stationary Rayleigh number $R_n$ is plotted against $N_A$ for fixed values of $N_A = 5, \varepsilon = .1, k_1 = 1, L_e = 5$ and different values of $R_{\sigma_n} = 50, 100, 150, \gamma = 1, 2, 3$ The Rayleigh number increases with increases in $N_A$, which shows modified diffusivity number $N_A$ has stabilizing effect on the stationary deformation.
Figure 5 represents the variation of stationary Rayleigh number with nanoparticle Rayleigh number $R_{a_n}$ for different values of $L_e$. The stationary Rayleigh number $R_a$ is plotted against $R_{a_n}$ for fixed values of $p_l = 5$, $\varepsilon = .1$, $k_1 = 1$, $R_{a_n} = 50$, $\gamma = 1$ and different values of $N_A = 5, 10, 15$, $L_e = 5, 10, 15$. The Rayleigh number increases with increases in $R_{a_n}$, which shows nanoparticle Rayleigh number $R_{a_n}$ has stabilizing effect on the stationary deportation.

Figure 6 represents the variation of stationary Rayleigh number with couple-stress parameter $\gamma$ for different values of $L_e$. The stationary Rayleigh number $R_a$ is plotted against $\gamma$ for fixed values of $p_l = 5$, $\varepsilon = .1$, $k_1 = 1$, $R_{a_n} = 50$, $\gamma = 1$ and different values of $N_A = 5, 10, 15$, $L_e = 5, 10, 15$. The Rayleigh number decreases with increases in $\gamma$, which shows couple-stress has destabilizing effect on the stationary deportation.

Figure 7 represents the variation of stationary Rayleigh number with couple-stress parameter $\gamma$ for different values of $\gamma$. The stationary Rayleigh number $R_a$ is plotted against $\gamma$ for fixed values of $N_A = 5$, $\varepsilon = .1$, $k_1 = 1$, $L_e = 5$ and different values of $p_l = 5, 10, 15$, $R_{a_n} = 1, 2, 3$ The Rayleigh number decreases with increases in $\gamma$, which shows couple-stress has destabilizing effect on the stationary deportation.
Fig. 2: Variations of stationary Rayleigh number with Lewis number

Fig. 3: Variations of stationary Rayleigh number with Modified diffusivity ratio number
Fig. 4: Variations of stationary Rayleigh number with Modified diffusivity ratio number

Fig. 5: Variations of stationary Rayleigh number with nanoparticle Rayleigh number
7. CONCLUSIONS

The effect of couple-stress on thermal impermanence of visco-elastic Walter’s (modal B’) nanofluid layer through a porous medium is investigated by using linear instability analysis. The main conclusions from the analysis of this paper are as follows:

(1) For the stationary convection couple-stress has destabilizing effect on the system.

(2) Lewis number, modified diffusivity ratio and nano particle Rayleigh number have stabilizing effect on the stationary convection.

8. NOMENCLATURE

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<th>Symbol</th>
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<th>Greek Symbols</th>
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<tr>
<td>a</td>
<td>dimensionless resultant wave number</td>
<td></td>
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<td>$d'$</td>
<td>Thickness of nanofluid layer</td>
<td>$\alpha$</td>
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**Symbols and Definitions:**

- $D_B$: Brownian diffusion coefficient
- $D_T$: Thermophoretic diffusion coefficient
- $\rho$: Density of nanofluid
- $g$: acceleration due to gravity
- $\gamma$: Modified couple stress parameter
- $n$: growth rate of disturbances
- $k_1$: Medium permeability
- $q_f$: Velocity vector
- $R_a$: Rayleigh number
- $R_{a_m}$: Density Rayleigh number
- $R_{a_n}$: Nano particle Rayleigh number
- $T$: Temperature
- $T_1$: Reference temperature
- $t$: time
- $P_r$: Prandtl number
- $\delta$: Fluid viscosity
- $\delta'$: Couple-stress fluid viscosity
- $L_e$: Lewis number
- $N_A$: Modified diffusivity ratio
- $N_B$: Modified particle-density increment
- $\mu$: Viscosity
- $\epsilon$: Porosity
- $p$: Hydrostatic pressure
- $\mu'$: Kinematic visco-elasticity
- $(\rho_c)_p$: Heat capacity nanoparticles
- $(\rho_c)_f$: Heat capacity of base fluid
- $\varphi$: volume fraction nanoparticle
- $\rho_p$: density of nanoparticles
- $\rho_f$: density of base fluid
- $k$: Thermal diffusivity
- $v$: Kinematic Viscosity
- $v'$: Kinematic visco-elasticity

**Superscripts:**

- '': non-dimensionless variables
- ‘’ perturbed quantities

9. REFERENCES:


