Production Inventory Model under Time Dependent Demand, UnevenManufacturing Rate and Manufacture Time Dependent Selling Price

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Abstract

In this paper, an inventory model for production of a singlearticle with anunevenmanufacturing rate and manufacturing time subsidiary selling cost has been considered. The considered production inventory model is accepted to create perfect items in beginning however because of different elements, after some time the production begins diminishing exponentially with time, i.e., the variable production rate has been thought of. The demand is time subordinate. Initially up to certain time, production rate remains constant. But after some time, due to various factors, production will decrease. Therefore, the efficiency (E) of such factors must be increased to get more production which can maintain the production efficiency cost which has been applied. Considering this fact inverse efficiency λ has been introduced in production rate.By utilizing differential calculus, expected maximumprofit has been resolved. The goal of the examination is to decide the ideal arrangement for a production framework that expands the total benefit subject to certain limitations viable. Results are examined by means of a mathematical example to outline the hypothesis.

Keywords:inventory optimization,variable production rate, mathematical modeling, repaired items.

1. Introduction

In assembling and trade activity, the inventory issues are normal components. In any manufacture inventory framework, vulnerabilities are generally connected with demand, crude materials flexibly, different relevant expenses, and life of the manufacturingamenities, appliance fixes and repairs time. At the point when these vulnerabilities are not important then,using old style "Economic Order Quantity" (EOQ) or "Economic Production Quantity" (EPQ) modellings, we can assessed these vulnerabilities.

At the beginning when production is going on in a manufacturing plant, production rate remain practically consistentup to certain time. Since all elements related with manufacturing are afresh, for example, all equipments are in good conditions, all specialists are physically and mentally sound. For the most part as time of manufacturing builds, all factors identified with the manufacture procedure go through some sluggishness i.e., there exist, after definitemoment, wasteful aspects of all variables engaged with the manufacturing, leading to the diminishing of the production. Presently if there is not any course of action for appropriate manufacturer to satisfy the client complete demand, supply of articles would be in deficiency, i.e., at last client demands as time advances, won't be satisfied. Because any maker won't anticipate such circumstance, therefore in such conditions manufacturer takes definite choices to expand the effectiveness of all elements to create and to keep up the proper production to fulfill the clients' order paying several additional costs,recognized as proficiency cost, E.

Initially up to certain time, production rate remains constant. But after some time, due to various factors, production will decrease. Therefore, the efficiency (E) of such factors must be increased to get more production which can maintain the production efficiency cost which has been applied. Considering this fact inverse efficiency λ has been introduced in production rate.

Classical economic production quantity (EPQ) model expects the reliability of the production units. This notion, though, doesn't qualify for some actual systems. Indeed, even the most excellent and the most advanced manufacturing systems go through the circumstance of unexpected emergency like machine breakdown, labor-strike etc, and the time taken in overcoming the problem varies with type of problem. Production capacity of the production unit may suffer due to these and unavailability of skilled or unskilled workers. Generally, the manufacturing system is considered as adaptable to deliver according to the demand. The manufacturing may stop at any arbitrary time and this period isadditionally thought to be stochastic. The purpose of this investigation is to decide the expected optimum production run time with the end goal of reducing the overall cost per unit time. A lot of work has been done related to these issues and offered solutions accordingly. Teng and Chang [1] considered EPQ model of deteriorating item with cost and inventory. Hou[2] considered a model of inflation with shortages, stock dependent demand.Sana *et al.*[3] considered a EPQ model having trended demand with shortages and deterioration. Chakraborty*et al.*[4] considered an EPQ with machine breakdown and deterioration. They considered preventive and corrective maintenance, simultaneously. An imperfect multi-product manufacturing system with variable demand is investigated. The study proposes an optimal production policy to reducethe failure rate and energy consumption of the production system with an additional development cost Singh and Jain [5] considered an EPQ model with inflation and supplier credits. Sana [6] considered a model to evaluate the rate and consistency of product to maximize the profit. Giri and Chakraborty[7] proposed a model for vendor and buyer supply chain coordination and developed a screening after each replenishment to optimize the cost. Wang *et al.* [8] developed an EOQ model to optimize the function with defective quality stuff, and decision variables depend upon time interval. Li *et al.* [9] considered for random demand and remanufacturing yields to optimize the model. Marchi*et al.*[10] considered the learning outcome in power efficiency in manufacturing companies. Therefore, they proposed a lot-sizing problem to illustrate the interaction between learning in production and energy efficiency directly and indirectly and also an appropriate decision about the lot size quantity. Shamayleh*et al.* [11] proposed a replenishment strategy for retailers who is facing a time-varying demand for cold products Singh *et al.* [12] considered an EPQ model to examine the preservation technology impact with machine breakdown by assuming multivariate demand rate with crispy and fuzzy situation.

2. AssumptionsMade for Proposed Model

To show the planned model the accompanying assumptions have been utilized:

(i) The demand is time subordinate and is given by $\alpha+\beta t$, $0<\beta<1$, where β signifies the shape parameter and is a proportion of responsiveness of demand and α means the scale parameter.

(ii) S: Total set up cost

(iii) P.R.: production rate per unit time

(iv) I: inventory of the production

(v) T: Total selling period

(vi) S.P.: producer selling cost per product

(vii) λ: inverse efficiency (choice variable)

(viii) K: total production cost

(x) h: holding cost per unit product per unit time

(xi) R: Total revenue

(xii) E: Efficiency cost

(xiii) H: total holding cost

(xv) T.P.: Total Profit

(xvi) C_e: rate of efficiency cost

(xvii) t_1: time period of constant production

(xviii) t_2: total production period

(xix) Q_1: inventory level at time t_1

(xx) Q_2: inventory level at time t_2

At the start of any production period the pace of manufacturing will be equal for some extent. After that the manufacturing rate will diminish because of a few inherent issues in the framework, for example, hardware deficiency, work issues, inadequacy of crude materials and so on. To satisfy the clients' demand during the selling time frame T, a few efficiencies (E) of various factors in the framework must be expanded for additionalmanufacturing. Considering this reality in the manufacture inventory framework, production rate, PR, taken as a component of another variable λ identified as inverse efficiency, is planned as follows:

$$PR = p, \qquad 0 \le t \le t_1$$

$$PR = pe^{(-\lambda(t-t_1))} t_1 \le t \le t_2$$

$$Where\lambda = \frac{1}{E}$$

The production will be halted after a specific time t_2 so that the framework gives the ideal benefit fulfilling the clients' all out demand.

Our objective is to find time at which producer stop the production to get the maximum profit. Demand is taken as time dependent.In this paper it is accepted that the selling cost of a thing relies on the manufacture time, since at first in the fabrication everything is new, so each created thing will be of satisfactory quality. Be that as it may, after certain time, the nature of the thing will be decayed progressively regarding time up to the finish of production because of hardware flaw, absence of expertsor the quality people for continual working, and so forth. In this way the selling cost S.P. of a thing has been assumed the premise of production time, as per the accompanying:

$$S.P. = C \qquad 0 \le t \le t_1$$

$$S.P. = Cxt_1 \leq t \leq t_2 \qquad ,0 < x < 1$$

3. Formulation of Mathematical Model for Proposed Inventory System

Initially system produces perfect item i.e. from time t=0 to t=t_1. Production rate P remains constant. At t=t_1 inventory level is Q_1. After time$t_1$manufacture rate decreases exponentially andmanufacturing stops at time t=t_2. Inventory built up through the period $[0, t_2]$ to satisfy the demand. For the period of the period [t_2,T] stockslowly decreases and it depletes at the end of period t=T. Let I be stock level at the time *t*, then differential equation of *I* is

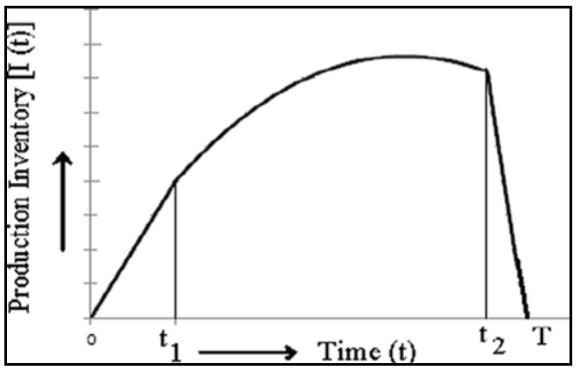

Fig.1 Graphical illustration of planned model

$$\frac{dI}{dt} = P - (\alpha + \beta t) \qquad ,0 \leq t \leq t_1 \qquad (1)$$

$$\frac{dI}{dt} = Pe^{-\lambda(t-t_1)} - (\alpha + \beta t) \qquad ,t_1 \leq t \leq t_2 \qquad (2)$$

$$\frac{dI}{dt} = -(\alpha + \beta t) \qquad ,t_2 \leq t \leq T \qquad (3)$$

Using the boundary condition

$I(0) = 0,\ I(t_1) = Q_1\ ,I(t_2) = Q_2\ ,\ I(T) = 0$

(4)

Integration of the differential equation (1) for the interval [0, t] yields

$$\int_0^t dI = \int_0^t [P - (\alpha + \beta t)]dt$$

or$I(t) - I(0) = (P - \alpha)t - \frac{\beta t^2}{2}$

with boundary condition I(t_1)=Q_1,we have,

$$Q_{1=}(P - \alpha)t_1 - \frac{\beta t_1^2}{2} \qquad (5)$$

Again, integration of the differential equation (2) for the interval $[t_1,t]$ yields

$$\int_{t_1}^{t} dI = \int_{t_1}^{t} \left[Pe^{-\lambda(t-t_1)} - (\alpha + \beta t)\right] dt$$

$$I(t) - I(t_1) = \left[\frac{-Pe^{-\lambda(t-t_1)}}{\lambda} - \alpha t - \frac{\beta t^2}{2}\right]_{t_1}^{t}$$

Using the boundary condition $I(t_1) = Q_1$ and $I(t_2) = Q_2$

$$Q_2 = Q_1 - \frac{Pe^{\lambda t_1}(e^{-\lambda t_2} - e^{-\lambda t_1})}{\lambda} - \alpha(t_2 - t_1) - \frac{\beta(t_2^2 - t_1^2)}{2} \qquad (6)$$

Also, performing integration of the differential equation (3) for the interval $[t_2,t]$, it is obtained that

$$\int_{t_2}^{t} dI = \int_{t_2}^{t} -(\alpha + \beta t) dt$$

Or $$I(t) - I(t_2) = -\alpha(t - t_2) - \frac{\beta(t_2^2 - t_1^2)}{2}$$

Using boundary condition $I(T) = 0$ we have

$$Q_2 - \alpha(T - t_2) - \frac{\beta(T^2 - t_2^2)}{2} = 0 \qquad (7)$$

Using equations (3), (4) and (5) the time t_2 at which productions stop, is obtained as follows

$$t_2 = t_1 - \frac{\ln[1 + \lambda t_1 - \frac{\lambda(\alpha T + \frac{\beta T^2}{2}}{P}}{\lambda} \qquad (8)$$

Now the various costs related with the planned inventory method are production cost (K), setup cost (S), holding cost (H), and efficiency cost (E).

The expressions for these costs are obtained as follows:

Total holding cost "H":

$$H = h\int_{0}^{T} I dt$$

$$=h[\frac{(P-\alpha)t_1^2}{2}-\frac{\beta t_1^3}{6}+Q_1(t_2-t_1)+\frac{P\left(e^{-\lambda(t_2-t_1)}-1\right)}{\lambda^2}+\frac{P(t_2-t_1)}{\lambda}-\frac{\alpha(t_2^2-t_1^2}{2}-\frac{\beta(t_2^3-t_1^3)}{6}+$$
$$\alpha t_1(t_2-t_1)+\frac{\beta t_1^2(t_2-t_1)}{2}+Q_2(T-t_2)-\frac{\alpha(T^2-t_2^2)}{2}+\alpha t_2(T-t_2)-\frac{\beta(T^3-t_2^3)}{6}+\frac{\beta t_2^2(T-t_2)}{2}] \quad (9)$$

And the efficiency cost “E” is obtained as:

$$\text{E} = c_e\int_{t_1}^{t_2} Pe^{-\lambda(t-t_1)}\,dt$$

$$=\frac{c_eP(1-e^{-\lambda(t_2-t_1)})}{\lambda} \quad (10)$$

Now total Revenue “R” obtained by selling all items to the customers at the rate of “p” per item is given by

$$\text{R}=\int_0^{t_1} Pcdt+\int_{t_1}^{t_2} Px\,e^{-\lambda(t-t_1)}dt$$

$$=Pct_1-\frac{Pxc\,(e^{-\lambda(t_2-t_1)}-1)}{\lambda} \quad (11)$$

Therefore,total profit satisfying in the production inventory system is given by

$$TP(\lambda,t_2)=R-K-S-H-E$$

$$TP(\lambda,t_2)=$$
$$Pct_1-\frac{Pxc\,(e^{-\lambda(t_2-t_1)}-1)}{\lambda}-K-S-h\left[\frac{(P-\alpha)t_1^2}{2}-\frac{\beta t_1^3}{6}+Q_1(t_2-t_1)+\frac{P\left(e^{-\lambda(t_2-t_1)}-1\right)}{\lambda^2}+\frac{P(t_2-t_1)}{\lambda}-\right.$$
$$\frac{\alpha(t_2^2-t_1^2}{2}-\frac{\beta(t_2^3-t_1^3)}{6}+\alpha t_1(t_2-t_1)+\frac{\beta t_1^2(t_2-t_1)}{2}+Q_2(T-t_2)-\frac{\alpha(T^2-t_2^2)}{2}+\alpha t_2(T-t_2)-$$
$$\left.\frac{\beta(T^3-t_2^3)}{6}+\frac{\beta t_2^2(T-t_2)}{2}\right]-\frac{c_eP(1-e^{-\lambda(t_2-t_1)})}{\lambda} \quad (12)$$

This equation represents the required deterministic profit function for time dependent demand.

4. Procedure for Optimal Result

Our purpose is to establish the optimal value of t_2, so that total profit (TP) can be maximized. The basic condition for TP to be maximum is $\frac{d(TP)}{dt_2}=0$ and $\frac{d^2TP}{dt_2^2}<0$

Now $\frac{d^2TP}{dt_2^2}=\frac{d^2R}{dt_2^2}-\frac{d^2K}{dt_2^2}-\frac{d^2S}{dt_2^2}-\frac{d^2H}{dt_2^2}-\frac{d^2E}{dt_2^2}$

Since $\frac{dt_1}{dt_2} = 1 + \frac{1-(\frac{\alpha+\beta T}{P})\frac{dT}{dt_2}}{\lambda t_1 - \frac{\lambda(\alpha T + \frac{\beta T^2}{2})}{P}}$

Therefore $\frac{d^2R}{dt_2^2} = \frac{d^2K}{dt_2^2} = \frac{d^2S}{dt_2^2} = 0$

This gives $\frac{d^2TP}{dt_2^2} = -\frac{d^2H}{dt_2^2} - \frac{d^2E}{dt_2^2}$

This has a negative value. Hence $\frac{d^2TP}{dt_2^2} < 0$

5. Numerical Example

An assembling organization needs to begin a business of creating mobiles in a territory where the greater part of the people groups live contingent upon development. Before the beginning of trade the organization overview the diverse correlated issues about the business, for example, demand of the item, financial state of individuals, land esteem, accessibility of neighborhood master works and atmosphere and so forth on that district. From the study it is realized that ordinarily in consistently principle crops are reaped enormously for starting a few months; however there are crops pretty much during the rest of the months of the year. Be that as it may, measure of gathered yields during the year thoroughly relies upon the atmosphere and it is seen that when climate is very kindness then the yields are collected massively.

During the time (0.4 year) of collecting people groups have a lot of cash and after that their income steadily diminishes. Based on this data organization has chosen to fix a consistent and high selling cost (40$) during the interim 0.4 year and afterward selling cost is decreased to 50%. Here organization needs to create the things to satisfy the demand for one year. Since at the hour of beginning of fabrication, all machinery and other related assets for manufacturing are in fine state, henceforth organization considers delivering the things at the rate 200 every year and after time interim 0.4 year there could happen deficiency in the machine differently, so production by machines regularly will be diminished exponentially. In the event that such circumstance emerges, at that point to satisfy the clients' complete demand organization wants to expand production by augmenting the effectiveness of the machine. Be that as it may, organization doesn't have the foggiest idea how much effectiveness is required to satisfy the fixed demand. Henceforth, under such conditions maker need to make out the normal most extreme benefit from the tradejudgment the ideal productivity of the machine and ideal time up to which production be proceeded.

Here,

p= 200; α= 20; t_1 = 0.4;C= 40;h = 1; C_e= 2;S = 90;X=0.5.

Table: 1below depicts the effect of responsiveness of demand on total production period, total selling period, inverse efficiency and total profit.

Table 1: Effect of β on t_2, T, λ and $T.P.$

β	**0.2**	**0.4**	**0.6**	**0.8**
t_2	0.933199	0.9786	1.0247	1.0718
T	8.933199	8.98	9.025	9.0718
λ	5.6264	5.1849	4.8023	4.4656
T.P.	1408.9211	1193.6542	1097.6440	1002.3530

Figure2 below shows the variation of total production period with respect to responsiveness of demand. The total production period increases with increase in responsiveness of demand.

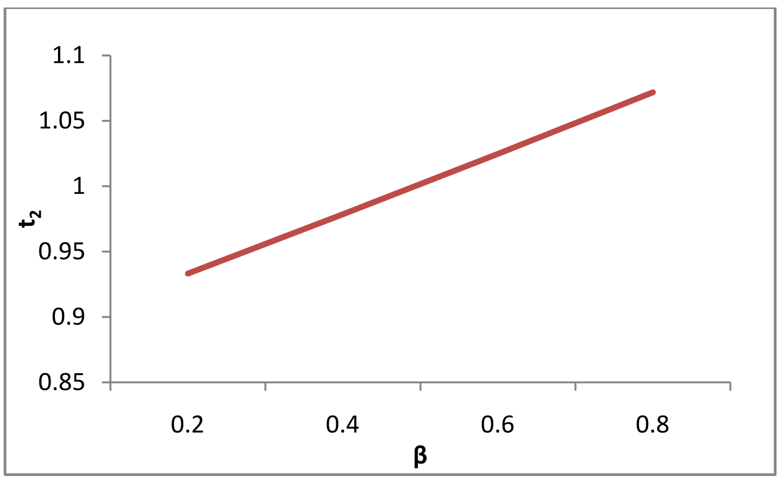

Fig.2 Effect of responsiveness of demand on total production period

Figure 3 shows the variation of total selling period with respect to responsiveness of demand. The total selling period also increases with increase in responsiveness of demand.

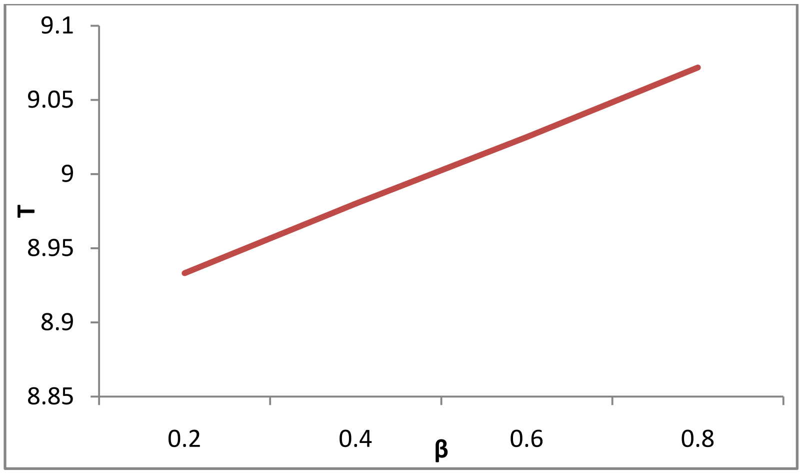

Fig.3 Effect of responsiveness of demand on total selling period

Figure 4 shows the variation of inverse efficiency with respect to responsiveness of demand. The inverse efficiency decreases with increase in responsiveness of demand.

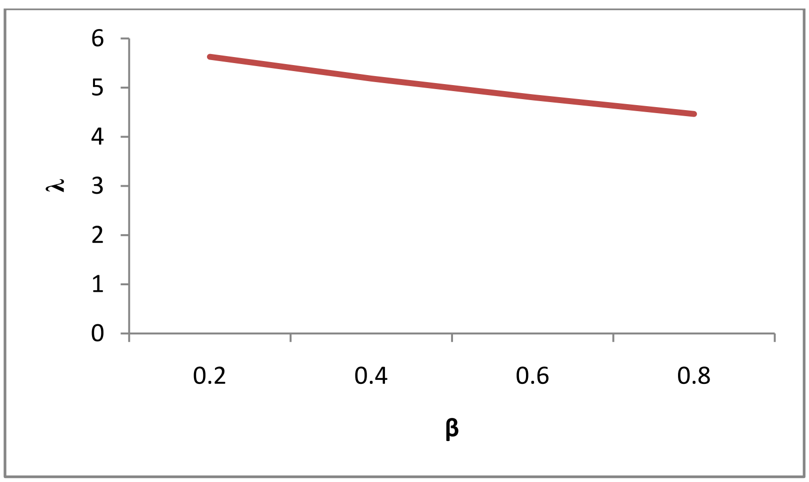

Fig. 4 Effect of responsiveness of demand on inverse efficiency

Figure 5 shows the increase in total profit with respect to responsiveness of demand. It can be seen from the graph that total profit increases with increase in responsiveness of demand.

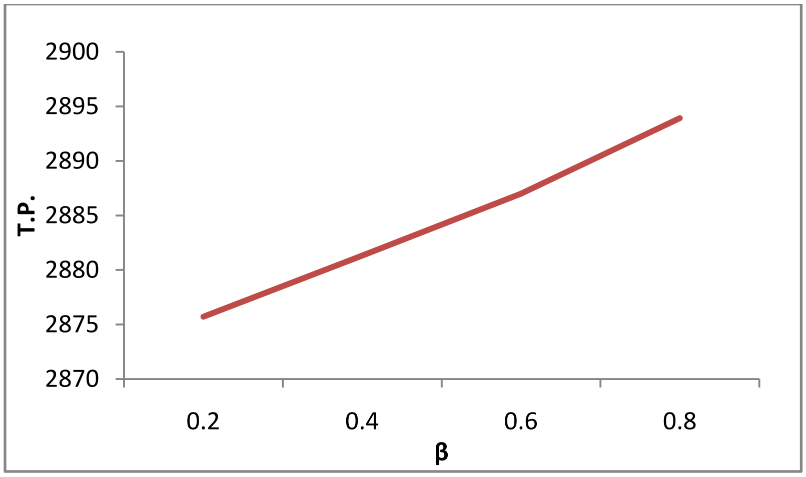

Fig.5 Effect of responsiveness of demand on total profit

6. Observations

- As responsiveness of demand, β, increases, inventory level increases, and the time at which the production stops also increases.
- The higher value of responsiveness of demand, β, leads to a decrease in inverse efficiency i.e., efficiency increases.
- As responsiveness of demand, β, increases, total profit also increases.

7. Conclusion

During the time (0.4 years) of harvesting people groups have a lot of cash and after that their income steadily diminishes. Based on this data organization has preferred to stickto a consistent and elevated selling cost (40$) through the interim 0.4 years and afterward selling cost is lowered to 50%. At the beginning of production all machinery and other associated assets for manufacturing are in good condition, henceforth organization considers delivering the things at the rate 200 every 0.4 year and past time interim of 0.4 years there may be deficiency in the machine differently, so manufacturing by machines will be diminished exponentially. In the event that such circumstance emerges, at that point to satisfy the clients' complete demand organization wants to expand production by augmenting the effectiveness of the machine. Henceforth, under these conditions the normal most extreme benefit from the tradejudgment the ideal productivity of the machine and ideal time up to which production proceeds was calculated by taking a particular case.

The followingare the finding of the case discussed:

1. The organization is selling its products in accordance with customer demand as well as their pocket.
2. By applying the extra efficiency,the organization is getting profit even if their production is exponentially decreasing.
3. Their work is going on in all the seasons.

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