An Approach for Solving Multi-Objective Linear Fractional Programming Problem with fully Rough Interval Coefficients

Mohamed Solomon [1], Hegazy Zaher [2], Naglaa Ragaa [3]

[1] Ph.D. Researcher, Department of Operations and Management Research, Faculty of Graduate Studies for Statistical Research, Cairo University, Egypt.

[2] Professor Doctor in Mathematical Statistics, Faculty of Graduate Studies for Statistical Research, Cairo University, Egypt.

[3] Professor Doctor in Operations Research, Faculty of Graduate Studies for Statistical Research, Cairo University, Egypt.

Abstract

In this paper, a multi-objective linear fractional programming (MOLFP) problem is considered where all of its coefficients in the objective function and constraints are rough intervals (RIs). At first to solve this problem, we will construct two MOLFP problems with interval coefficients. One of these problems is an MOLFP where all of its coefficients are upper approximations of RIs and the other is an MOLFP where all of its coefficients are lower approximations of RIs. Second, the MOLFP problems are transformed into a single objective linear programming (LP) problem using a proposal given by Nuran Guzel. Finally the single objective LP problem is solved by a regular simplex method which yields an efficient solution of the original MOLFP problem. A numerical example is given to demonstrate the results.

Keywords: rough set theory, multi-objective, interval coefficients, rough interval coefficients, multi-objective linear fractional programming, linear fractional programming.

1. Introduction

In several applications of nonlinear programming a function is to be maximized or minimized which involves one or several ratios of functions. Such optimization problems are commonly called fractional programs, abbreviating the term 'fractional functionals program' initially suggested by Charnes and Cooper [2] in their classic paper in 1962. Rarely the term 'hyperbolic program' is used as well. To improve the terminology one may think of the term 'ratio program.' However, such a change may not come easy after well over one thousand publications have appeared in this area of nonlinear programming.

Linear Fractional Programming (LFP) problem is a mathematical programming problem where the objective function is the ratio of two linear functions subject to the constraints with linear equalities or inequalities. The Hungarian mathematicians, Martos and Whinston, [4] developed linear fractional programming problem in the 1960s. LFP problem is applied when the constraints and objective functions are deterministic in nature.

Fractional programming involves the optimization of one or several ratios of functions subject to some linear restrictions. In literature, various methods can be observed to solve different models of linear fractional programming LFP problem. Among the solution methods, the transformation technique developed by Charnes and Cooper [2], the simplex based algorithm proposed by Swarup [13] is widely accepted. The simplex method for solving LFP problem described by Bajaliov [9] is similar to Swarup's method.

The Rough set theory approach has major importance in the areas of machine learning, knowledge acquisition, decision analysis, and knowledge discovery from a database [11]. It has been successfully applied in many real-world problems, counting decision algorithms [25], pharmacology [12], and civil engineering [17] and among others. at the current time, some papers have been established on rough programming [3]. Newly, a new kind of rough programming was suggested by Youness [8] and Osman et al. [15], where they defined two solutions, ideas as a surely optimal solution and possibly optimal solution.

In 2006, Robolledo [16] suggested RIs and then the rough intervals used to deal with partially unknown or ill-defined parameters and variables. RIs are presented to adjust the rough set principles to model continuous variables. RSTs were used only to handle discrete objects, initially, and could not represent continuous values. RI is a specific case of rough sets. It achieves all the rough sets' properties and basic concepts, including the upper and lower approximation definitions [16]. More details of RIs are stated in the next section.

In the modern age, some new approaches have also been reported to solve MOLFP and FMOLFP problems. Farhana Akond [10] developed a method for solving fuzzy multi-objective linear fractional programming FMOLFP problem. At first the FMOLFP problem is converted into (crisp) multi-objective linear fractional programming MOLFP problem using the graded mean integration, representation (GMIR) method proposed by Chen and Hsieh. That is, all the fuzzy parameters of FMOLFP problem are converted into crisp values. Then the MOLFP problem is transformed into a single objective linear programming LP problem using a proposal given by Nuran Guzel [18]. Finally the single objective LP problem is solved by a regular simplex method which yields an efficient solution of the original FMOLFP problem. Jain [22] proposed a method using Gauss elimination technique to derive a numerical solution of multi-objective linear programming (MOLP) problem. Then Jain [23] extended his work for MOLFP problem. Porchelvi et al. [20] presented procedures for solving both MOLFP problem and FMOLFP problem using the complementary development method of Dheyab [1], where the fractional linear programming is transformed into a linear programming problem. Guzel and Sivri [19] presented a method for finding an efficient solution of MOLFP problem using goal programming. Later Guzel [18] proposed a simplex type algorithm for finding an efficient solution of MOLFP problem based on a theorem studied in a work by Dinkelbach [24], where he converted the main problem into a single objective LP problem.

Ammar and muamer [6] proposed algorithm for solving fuzzy rough linear fractional programming problem, where all variables and coefficients are fuzzy rough number. After that they used the decomposition to the fuzzy linear fractional programming problem for obtaining an optimal fuzzy rough solution, based on the variable transformation method. Further, the proposed approach can be extended for solving FRLFP problem where all coefficients are trapezoidal fuzzy numbers. Then Ammar and muamer [7] extended their work by presented a new approach for solving multi-objective linear fractional programming with fuzzy rough coefficients (MOFRLFP) problem by two methods ($\alpha - cut$, ranking function). Later Mohamed S. Osman et al. [14] proposed approach to solve multi-level multi-objective fractional programming problem where some or all of its coefficients in the objective function are rough intervals. in the first phase of the solution approach and to avoid the complexity of the problem, two FP problems with interval coefficients will be constructed. At the second phase, a membership function was constructed to develop a fuzzy goal programming model for obtaining the satisfactory solution of the multi-level multi-objective fractional programming problem. In 2021, E. Fathy [5] presented a suitable solution procedure to solve the fully rough multi-objective multi-level linear fractional programming (FRMMFP) problem. First, an extension of interval method is presented to deal with the roughness of the stated problem. Then, an iterative technique is proposed for linearization of fractional objectives. Finally, a modification of fuzzy approach is provided in the environment of the fully rough to solve the linear model.

The motivation of our discussion in this paper is to improve a method to determine the optimal solution of an MOLFP problem with rough interval coefficients.

The rest of the paper is prepared as follows. In Section 2, some basic knowledge of RI`s are presented. In Section 3, an MOLFP problem is discussed. In Section 4, an MOLFP problem with rough interval coefficients is discussed. Section 5, proposed a solution method for an MOLFP problem with rough interval coefficients. In section 6, numerical example for illustrating the solution of proposed method. Finally, concluding remarks are given in Section 7.

2. Rough intervals

in this section, Some definitions and properties of rough intervals are given. [16]

Definition: The qualitative value A is called a rough interval when one can assign two closed intervals A_* and A^* on R to it where $A_* \subseteq A^*$. Moreover,

(a) If $x \in A_*$ then A surely takes x (denoted by $x \in A$).
(b) If $x \in A^*$ then A possibly takes x.
(c) If $x \notin A^*$ then A surely does not take x (denoted by $x \notin A$).

A_* and A^* are called the lower approximation interval (LAI) and the upper approximation interval (UAI) of A, respectively. Additional, A is denoted by $A = (A_*, A^*)$.
Note that the intervals A_* and A^* are not the complement of each other.

The arithmetic operations on RIs are based on interval arithmetic [16]. We will state some of these arithmetic operations as follows [12]:

Let $A = \left([\underline{a^l}, \underline{a^u}], [\overline{a^l}, \overline{a^u}] \right)$ and $B = \left([\underline{b^l}, \underline{b^u}], [\overline{b^l}, \overline{b^u}] \right)$ be two rough intervals. Then, we have:

[Addition] $A + B = \left([\underline{a^l} + \underline{b^l}, \underline{a^u} + \underline{b^u}], [\overline{a^l} + \overline{b^l}, \overline{a^u} + \overline{b^u}] \right)$.

[Subtraction] $A - B = \left([\underline{a^l} - \underline{b^u}, \underline{a^u} - \underline{b^l}], [\overline{a^l} - \overline{b^u}, \overline{a^u} - \overline{b^l}] \right)$.

[Negative] $-A = \left([-\underline{a^u}, \underline{-a^l}], [-\overline{a^u}, -\overline{a^l}] \right)$.

[Intersection] $A \cap B = ([\max\{\underline{a^l}, \underline{b^l}\}, \min\{\underline{a^u}, \underline{b^u}\}], [\max\{\overline{a^l}, \overline{b^l}\}, \min\{\overline{a^u}, \overline{b^u}\}]$.

[Union] $A \cup B = ([\min\{\underline{a^l}, \underline{b^l}\}, \max\{\underline{a^u}, \underline{b^u}\}], [\min\{\overline{a^l}, \overline{b^l}\}, \max\{\overline{a^u}, \overline{b^u}\}]$.

3. Multi-Objective Linear Fractional Programming Problem

An MOLFP problem is defined as follows

$$(\boldsymbol{MOLFP})\ \boldsymbol{Maximize}\ \{\boldsymbol{Z}(\boldsymbol{x}) = (\boldsymbol{z}_1(\boldsymbol{x}), \boldsymbol{z}_2(\boldsymbol{x}), \dots, \boldsymbol{z}_k(\boldsymbol{x}))\}$$
$$\boldsymbol{s.t}\quad \boldsymbol{Ax} \le \boldsymbol{b} \qquad (1)$$
$$\boldsymbol{x} \ge \boldsymbol{0}.$$

Where:

$\boldsymbol{S} = \{\boldsymbol{x} \epsilon \mathbb{R}^{\boldsymbol{n}} \mid \boldsymbol{Ax} \le \boldsymbol{b}, \boldsymbol{x} \ge \boldsymbol{0}, \boldsymbol{b} \epsilon \mathbb{R}^{\boldsymbol{m}}\}$, is the Feasible Set in Decision Space**.**

$\boldsymbol{A\ is\ an\ m \times n\ matrix,\ x} \in \mathbb{R}^{\boldsymbol{n}}\ \boldsymbol{and\ b} \in \mathbb{R}^{\boldsymbol{m}};\ (\boldsymbol{b} \ge \boldsymbol{0}), \boldsymbol{k} \ge \boldsymbol{2}.$

$$\boldsymbol{z}_i(\boldsymbol{x}) = \frac{\boldsymbol{c}_i^T \boldsymbol{x} + \boldsymbol{\alpha}_i}{\boldsymbol{d}_i^T \boldsymbol{x} + \boldsymbol{\beta}_i} = \frac{\boldsymbol{N}_i(\boldsymbol{x})}{\boldsymbol{D}_i(\boldsymbol{x})}\ ;\ \boldsymbol{c}_i^T, \boldsymbol{d}_i^T \epsilon \mathbb{R}^{\boldsymbol{n}}\ ;\ \boldsymbol{\alpha}_i, \boldsymbol{\beta}_i\ \epsilon \mathbb{R};\ for\ all\ i = 1,2,\dots,k$$

$$\boldsymbol{and}\ \boldsymbol{D}_i(\boldsymbol{x}) = \boldsymbol{d}_i^T \boldsymbol{x} + \boldsymbol{\beta}_i > \boldsymbol{0}, \qquad for\ all\ i = 1,2,\dots,k, \qquad for\ all\ x\ \epsilon\ S.$$

A solution $\overline{\boldsymbol{x}} \boldsymbol{\epsilon}\ \boldsymbol{S}$ is an efficient solution of the problem (MOLFP) if and only if there is no $\boldsymbol{x}\ \boldsymbol{\epsilon}\ \boldsymbol{S}\ such\ that\ \boldsymbol{z}_i(\boldsymbol{x}) \ge \boldsymbol{z}_i(\overline{\boldsymbol{x}})\ for\ all\ i = 1,2,\dots,k\ and\ \boldsymbol{z}_i(\boldsymbol{x}) > \boldsymbol{z}_i(\overline{\boldsymbol{x}})\ for\ at\ least\ one\ i.$ Note that, for vectors $an\ ;\ \boldsymbol{x} \ge \boldsymbol{y}\ implies\ \boldsymbol{x}_i \ge \boldsymbol{y}_i\ for\ each\ \boldsymbol{i}, \boldsymbol{x} \ge \boldsymbol{y}\ implies\ \boldsymbol{x}_i \ge \boldsymbol{y}_i\ for\ \boldsymbol{i}$ $and\ \boldsymbol{x}_r > \boldsymbol{y}_r\ for\ at\ least\ one\ \boldsymbol{i} = \boldsymbol{r}\ \ and\ \boldsymbol{x} > \boldsymbol{y}\ implies\ \boldsymbol{x}_i > \boldsymbol{y}_i\ for\ each\ i$.

4. MOLFP with Rough Interval Coefficients

In this section, Multi-objective linear fractional programming problem with rough interval coefficients (MOLFPRIC) is considered. Formulating an MOLFP model requires that Crisp values be selected for the model coefficients. The values of several of these coefficients are only approximately known. The major advantage of the proposed operations over the existing one is that algorithm deal with uncertainty coefficients which take the form of fully rough interval coefficients. Now, joining all the data in the MOLFP model is required.

Let us consider an MOLFPRIC as:

$$\max z_1 = \frac{\sum_{j=1}^{n}([\underline{c_{ij}^l}, \underline{c_{ij}^u}], [\overline{c_{ij}^l}, \overline{c_{ij}^u}])\, x_j + ([\underline{\alpha_{ij}^l}, \underline{\alpha_{ij}^u}], [\overline{\alpha_{ij}^l}, \overline{\alpha_{ij}^u}])}{\sum_{j=1}^{n}([\underline{d_{ij}^l}, \underline{d_{ij}^u}], [\overline{d_{ij}^l}, \overline{d_{ij}^u}])\, x_j + ([\underline{\beta_{ij}^l}, \underline{\beta_{ij}^u}], [\overline{\beta_{ij}^l}, \overline{\beta_{ij}^u}])}$$

$$\max z_2 = \frac{\sum_{j=1}^{n}([\underline{c_{ij}^l}, \underline{c_{ij}^u}], [\overline{c_{ij}^l}, \overline{c_{ij}^u}])\, x_j + ([\underline{\alpha_{ij}^l}, \underline{\alpha_{ij}^u}], [\overline{\alpha_{ij}^l}, \overline{\alpha_{ij}^u}])}{\sum_{j=1}^{n}([\underline{d_{ij}^l}, \underline{d_{ij}^u}], [\overline{d_{ij}^l}, \overline{d_{ij}^u}])\, x_j + ([\underline{\beta_{ij}^l}, \underline{\beta_{ij}^u}], [\overline{\beta_{ij}^l}, \overline{\beta_{ij}^u}])}$$

.
.
.

$$\max z_k = \frac{\sum_{j=1}^{n}(\left[\underline{c_{ij}^l}, \underline{c_{ij}^u}\right], [\overline{c_{ij}^l}, \overline{c_{ij}^u}])\, x_j + (\left[\underline{\alpha_{ij}^l}, \underline{\alpha_{ij}^u}\right], [\overline{\alpha_{ij}^l}, \overline{\alpha_{ij}^u}])}{\sum_{j=1}^{n}(\left[\underline{d_{ij}^l}, \underline{d_{ij}^u}\right], [\overline{d_{ij}^l}, \overline{d_{ij}^u}])\, x_j + (\left[\underline{\beta_{ij}^l}, \underline{\beta_{ij}^u}\right], [\overline{\beta_{ij}^l}, \overline{\beta_{ij}^u}])}$$

S.T

$$\sum_{j=1}^{n}(\left[\underline{a_{ij}^l}, \underline{a_{ij}^u}\right], [\overline{a_{ij}^l}, \overline{a_{ij}^u}])\, x_j \le [\underline{b_i}^l, \underline{b_i}^u], [\overline{b_i}^l, \overline{b_i}^u] \qquad i = 1, 2, \dots, m$$

$$x_j \ge 0 \qquad j = 1, 2, \dots, n \qquad (2)$$

where

$\left(\left[\underline{\boldsymbol{c}_{ij}^l}, \underline{\boldsymbol{c}_{ij}^u}\right], \left[\overline{\boldsymbol{c}_{ij}^l}, \overline{\boldsymbol{c}_{ij}^u}\right]\right)$, $([\underline{\boldsymbol{d}_{ij}}^l, \underline{\boldsymbol{d}_{ij}}^u], \left[\overline{\boldsymbol{d}_{ij}}^l, \overline{\boldsymbol{d}_{ij}}^u\right])$ are rough intervals coefficients of the objective function $(j = 1, 2, \dots, n \;, i = 1, 2, \dots, m)$,

$(\left[\underline{\alpha_{ij}^l}, \underline{\alpha_{ij}^u}\right], [\overline{\alpha_{ij}^l}, \overline{\alpha_{ij}^u}])$ are rough intervals constants of the numerator,

$(\left[\underline{\beta_{ij}^l}, \underline{\beta_{ij}^u}\right], [\overline{\beta_{ij}^l}, \overline{\beta_{ij}^u}])$ are rough intervals constants of the denominator,

$[\underline{a_{ij}}^l, \underline{a_{ij}}^u], [\overline{a_{ij}}^l, \overline{a_{ij}}^u]$ **,** $[\underline{b_i}^l, \underline{b_i}^u], [\overline{b_i}^l, \overline{b_i}^u]$ are rough interval coefficients of the constraints.

$x = (x_1, x_2, \dots, x_n)^T$ denote the vector of all decision variables

5. Our Proposed Approach for Solving MOLFPRIC

We first find the possibly optimal range $[\overline{z_1}^{pl}, \overline{z_1}^{pu}], [\overline{z_2}^{pl}, \overline{z_2}^{pu}]$ by solving MOLFP with the interval coefficients problem. Secondly, we find the surly optimal range $\left[\underline{z_1}^{sl}, \underline{z_1}^{su}\right], \left[\underline{z_2}^{sl}, \underline{z_2}^{su}\right]$ by solving MOLFP with the interval coefficients problem. Then we transform MOLFP problems into linear programming problems using Guzel's proposal [18]. Finally, the linear programming problems are solved by simplex method, whose optimal solution is the required efficient solution of the original problem.

Input: Consider an MOLFPRIC problem illustrated in the model (2)

Step1: Find the possibly optimal range $[\overline{z_1}^{pl}, \overline{z_1}^{pu}], [\overline{z_2}^{pl}, \overline{z_2}^{pu}]$ by solving the following MOLFP with interval coefficients problem [16]:

$$\max z_1 = \frac{\sum_{j=1}^{n}[\overline{c_{ij}^l}, \overline{c_{ij}^u}]\; x_j + [\overline{\alpha_{ij}^l}, \overline{\alpha_{ij}^u}]}{\sum_{j=1}^{n}[\overline{d_{ij}^l}, \overline{d_{ij}^u}]\; x_j + [\overline{\beta_{ij}^l}, \overline{\beta_{ij}^u}]}$$

$$\max z_2 = \frac{\sum_{j=1}^{n}[\overline{c_{ij}^{l}}, \overline{c_{ij}^{u}}]\ x_j + [\overline{\alpha_{ij}^{l}}, \overline{\alpha_{ij}^{u}}]}{\sum_{j=1}^{n}[\overline{d_{ij}^{l}}, \overline{d_{ij}^{u}}]\ x_j + [\overline{\beta_{ij}^{l}}, \overline{\beta_{ij}^{u}}]}$$

S.T

$$\sum_{j=1}^{n}\left(\left[\underline{a_{ij}^{l}}, \underline{a_{ij}^{u}}\right], [\overline{a_{ij}^{l}}, \overline{a_{ij}^{u}}]\right) x_j \le [\underline{b_i{}^{l}}, \underline{b_i{}^{u}}], [\overline{b_i{}^{l}}, \overline{b_i{}^{u}}] \qquad i = 1, 2, \dots, m$$

$$x_j \ge 0 \qquad j = 1, 2, \dots, n \qquad (3)$$

If the problem (3) is infeasible go to step 3.

Step 2: Find the surely optimal range $\left[\underline{z_1}^{sl}, \underline{z_1}^{su}\right], \left[\underline{z_2}^{sl}, \underline{z_2}^{su}\right]$ by solving the following MODNP with interval coefficients problem [16]:

$$\max z_1 = \frac{\sum_{j=1}^{n}\left[\underline{c_{ij}^{l}}, \underline{c_{ij}^{u}}\right]\ x_j + \left[\underline{\alpha_{ij}^{l}}, \underline{\alpha_{ij}^{u}}\right]}{\sum_{j=1}^{n}\left[\underline{d_{ij}^{l}}, \underline{d_{ij}^{u}}\right]\ x_j + \left[\underline{\beta_{ij}^{l}}, \underline{\beta_{ij}^{u}}\right]}$$

$$\max z_2 = \frac{\sum_{j=1}^{n}\left[\underline{c_{ij}^{l}}, \underline{c_{ij}^{u}}\right]\ x_j + \left[\underline{\alpha_{ij}^{l}}, \underline{\alpha_{ij}^{u}}\right]}{\sum_{j=1}^{n}\left[\underline{d_{ij}^{l}}, \underline{d_{ij}^{u}}\right]\ x_j + \left[\underline{\beta_{ij}^{l}}, \underline{\beta_{ij}^{u}}\right]}$$

S.T

$$\sum_{j=1}^{n}\left(\left[\underline{a_{ij}^{l}}, \underline{a_{ij}^{u}}\right], [\overline{a_{ij}^{l}}, \overline{a_{ij}^{u}}]\right) x_j \le [\underline{b_i{}^{l}}, \underline{b_i{}^{u}}], [\overline{b_i{}^{l}}, \overline{b_i{}^{u}}] \qquad i = 1, 2, \dots, m$$

$$x_j \ge 0 \qquad j = 1, 2, \dots, n \qquad (4)$$

Step 3: Transform MOLFP with interval coefficients (possibly optimal range) $[\overline{z_1}^{pl}, \overline{z_1}^{pu}], [\overline{z_2}^{pl}, \overline{z_2}^{pu}]$ to two models where their feasible set are $\overline{U^L}$, $\overline{U^U}$, respectively.

Step 4: Transform MOLFP with interval coefficients (surly optimal range) $\left[\underline{z_1}^{sl}, \underline{z_1}^{su}\right], \left[\underline{z_2}^{sl}, \underline{z_2}^{su}\right]$ to two models where their feasible set are $\underline{U^L}, \underline{U^U}$ respectively.

Step 5: Solve the problem $\overline{U^U}$, and obtain the upper bound $[\overline{z_1}^{pu}, \overline{z_2}^{pu}]$ by using Guzel's proposal.

Step 6: Solve the problem $\overline{U^L}$, and obtain the lower bound $[\overline{z_1}^{pl}, \overline{z_2}^{pl}]$ by using Guzel's proposal.

Step 7: Solve the problem $\underline{U^U}$ and obtain the upper bound $\left[\underline{z_1}^{su}, \underline{z_2}^{su}\right]$ by using Guzel's proposal.

Step 8: Solve the problem $\underline{U}^L$ and obtain the lower bound $\left[\underline{z_1}^{sl}, \underline{z_2}^{sl}\right]$ by using Guzel's proposal.

Step 9: There are three potential outcomes for MOLFPRIC Problem (2) as follows:

1) If MOLFP with interval coefficients Problems (3) and (4) have optimal ranges, then MOLFPRIC Problem (2) has a rough range as $\left(\left[\underline{z_1}^{sl}, \underline{z_1}^{su}\right], \left[\overline{z_1}^{pl}, \overline{z_1}^{pu}\right]\right), \left(\left[\underline{z_2}^{sl}, \underline{z_2}^{su}\right], \left[\overline{z_2}^{pl}, \overline{z_2}^{pu}\right]\right)$.
2) If MOLFP with interval coefficients Problem (4) has unbounded range, then MOLFPRIC Problem (2) has unbounded range.
3) If MOLFP with interval coefficients Problem (3) is infeasible, then MOLFPRIC Problem (2) is infeasible.

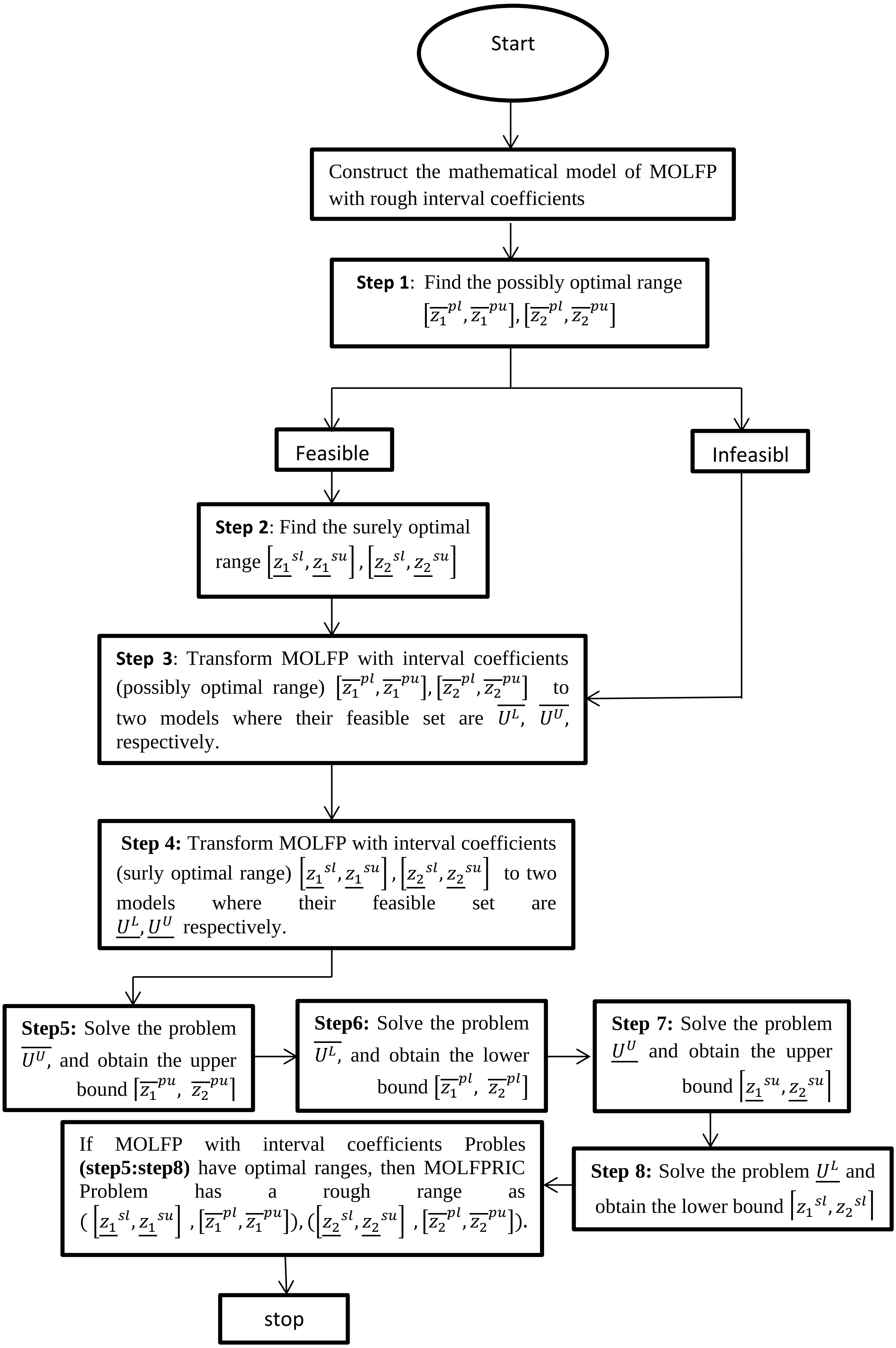

Figure 1: Flow chart to solve MOLFPRIC problems

6. Numerical Example

Consider the following multi-objective linear fractional programming problem with rough interval coefficients. (we modified This example in [10] with new assumptions)

$$\min z_1(x) = \frac{[-3,-1][-4,-1]x_1 + [1,3][0.5,4]x_2 + [1,2][0.5,3]}{[0.5,1][0.25,1.5]x_1 + [1,2][0.5,2.5]x_2 + [0.5,1][0.25,1.25]}$$

$$\min z_2(x) = \frac{[2,5][1,6]x_1 + [1,2][0.5,3]x_2 + [1,2][0.5,3]}{[1,2][0.5,2.5]x_1 + [1,3][0.5,4]x_2 + [0.5,1][0.25,1.25]}$$

s.t

$$[1,2][0.5,2.5]x_1 + [0.5,1][0.25,1.5]x_2 \leq [2,4][1,5]$$

$$[2,3][1,4]x_1 - [3,2][4,1]x_2 \leq [2,5][1,6] \qquad (5)$$

$$[0.5,1][0.25,1.75]x_1 + [1,2][0.5,2.5]x_2 \leq [1,3][0.5,3.5]$$

$$[0.5,1][0.25,1.5]x_1 + [2,3][1,4]x_2 \leq [1,2][0.5,3.5]$$

$$x_1, x_2 \geq 0$$

To solve Problem (5), we have to solve two MOLFP with interval coefficients problems as follows:

Step1:

$$\min z_1(x) = \frac{[-4,-1]x_1 + [0.5,4]x_2 + [0.5,3]}{[0.25,1.5]x_1 + [0.5,2.5]x_2 + [0.25,1.25]}$$

$$\min z_2(x) = \frac{[1,6]x_1 + [0.5,3]x_2 + [0.5,3]}{[0.5,2.5]x_1 + [0.5,4]x_2 + [0.25,1.25]}$$

s.t

$$[0.5,2.5]x_1 + [0.25,1.5]x_2 \leq [1,5]$$

$$[1,4]x_1 - [4,1]x_2 \leq [1,6] \qquad (6)$$

$$[0.25,1.75]x_1 + [0.5,2.5]x_2 \leq [0.5,3.5]$$

$$[0.25,1.5]x_1 + [1,4]x_2 \leq [0.5,3.5]$$

$$x_1, x_2 \geq 0$$

Step 2:

$$\min f_1(x) = \frac{[-3,-1]x_1 + [1,3]x_2 + [1,2]}{[0.5,1]x_1 + [1,2]x_2 + [0.5,1]}$$

$$\min f_2(x) = \frac{[2,5]x_1 + [1,2]x_2 + [1,2]}{[1,2]x_1 + [1,3]x_2 + [0.5,1]}$$

s.t

$$\begin{aligned} [1,2]x_1 + [0.5,1]x_2 &\le [2,4] \\ [2,3]x_1 - [3,2]x_2 &\le [2,5] \qquad (7) \\ [0.5,1]x_1 + [1,2]x_2 &\le [1,3] \\ [0.5,1]x_1 + [2,3]x_2 &\le [1,2] \\ x_1, x_2 &\ge 0 \end{aligned}$$

Step 3: The MOLFPIC Problem (6) is transformed to MOLFP problems R1 and R2, where their feasible sets are $\overline{U}^l$ *and* $\overline{U}^u$, respectively.

Step4: The MOLFPIC Problem (7) is transformed to MOLFP problems R3 and R4, where their feasible sets are $\underline{U}^l$ *and* $\underline{U}^u$, respectively.

R1: $\overline{z_1}^l = \min \frac{-4x_1 + 0.5x_2 + 0.5}{0.25x_1 + 0.5x_2 + 0.5}$ $\overline{z_2}^l = \min \frac{x_1 + 0.5x_2 + 0.5}{0.5x_1 + 0.5x_2 + 0.25}$ s.t $2.5x_1 + 1.5x_2 \le 1$ $4x_1 - x_2 \le 1$ $1.75x_1 + 2.5x_2 \le 0.5$ $1.5x_1 + 4x_2 \ge 0.5$ $x_1, x_2 \ge 0$	R2: $\overline{z_1}^u = \min \frac{-x_1 + 4x_2 + 3}{1.5x_1 + 2.5x_2 + 1.25}$ $\overline{z_2}^u = \min \frac{6x_1 + 3x_2 + 3}{2.5x_1 + 4x_2 + 1.25}$ s.t $0.5x_1 + 0.25x_2 \le 5$ $x_1 - 4x_2 \le 6$ $0.25x_1 + 0.5x_2 \le 3.5$ $0.25x_1 + x_2 \ge 3.5$ $x_1, x_2 \ge 0$
R3: $\underline{z_1}^l = \min \frac{-3x_1 + x_2 + 1}{0.5x_1 + x_2 + 0.5}$ $\underline{z_2}^l = \min \frac{2x_1 + x_2 + 1}{x_1 + x_2 + 0.5}$ s.t $2x_1 + x_2 \le 2$ $x_1 - 2x_2 \le 2$ $x_1 + 2x_2 \le 1$ $x_1 + 3x_2 \ge 1$ $x_1, x_2 \ge 0$	R4: $\underline{z_1}^u = \min \frac{-x_1 + 3x_2 + 2}{x_1 + 2x_2 + 1}$ $\underline{z_2}^u = \min \frac{5x_1 + 2x_2 + 2}{2x_1 + 3x_2 + 1}$ s.t $x_1 + 0.5x_2 \le 4$ $2x_1 - 3x_2 \le 5$ $0.5x_1 + x_2 \le 3$ $0.5x_1 + 2x_2 \ge 2$ $x_1, x_2 \ge 0$

Step 5: Solve the problem $\overline{U^U}$, and obtain the upper bound $\left[\overline{z_1}^{pu}, \overline{z_2}^{pu}\right]$ (R2) by using Guzel's proposal.

$$\overline{z_1}^u = \min \frac{-x_1 + 4x_2 + 3}{1.5x_1 + 2.5x_2 + 1.25}$$

$$\overline{z_2}^u = \min \frac{6x_1 + 3x_2 + 3}{2.5x_1 + 4x_2 + 1.25}$$

s.t

$$0.5x_1 + 0.25x_2 \leq 5$$

$$x_1 - 4x_2 \leq 6$$

$$0.25x_1 + 0.5x_2 \leq 3.5$$

$$0.25x_1 + x_2 \geq 3.5$$

$$x_1, x_2 \geq 0$$

To solve this MOLFP problem, we find the optimal value of each of the objective functions $\overline{z_1}^u$, $\overline{z_2}^u$ subject to the above constraints, using any of the methods for solving linear fractional programming problems (we used the equivalent of Charnes A. and Cooper W.W. [2]).

We get,

$\min \overline{z_1}^u = -0.10$
$\min \overline{z_2}^u = 0.82$

An LP problem, which is equivalent to the MOLFP problem, is constructed according to the proposed algorithm as follows:

$$\min \overline{U^U} = (-x_1 + 4x_2 + 3) + (0.10)(1.5x_1 + 2.5x_2 + 1.25) + (6x_1 + 3x_2 + 3) - (0.82)(2.5x_1 + 4x_2 + 1.25)$$

$$= 3.1x_1 + 3.97x_2 + 5.1$$

s.t

$$0.5x_1 + 0.25x_2 \leq 5$$

$$x_1 - 4x_2 \leq 6$$

$$0.25x_1 + 0.5x_2 \leq 3.5$$

$$0.25x_1 + x_2 \geq 3.5$$

$$x_1, x_2 \geq 0$$

the resulting $\overline{x_{1opt}}^u, \overline{x_{2opt}}^u = (0, 3.5)$ and $\overline{z_{1opt}}^u, \overline{z_{2opt}}^u = (1.7, 0.89)$

Step 6: Solve the problem $\overline{U^L}$, and obtain the lower bound $\left[\overline{z_1}^{pl},\ \overline{z_2}^{pl}\right]$ (R1) by using Guzel's proposal.

$$\overline{z_1}^{l} = \min \frac{-4x_1 + 0.5x_2 + 0.5}{0.25x_1 + 0.5x_2 + 0.5}$$

$$\overline{z_2}^{l} = \min \frac{x_1 + 0.5x_2 + 0.5}{0.5x_1 + 0.5x_2 + 0.25}$$

s.t

$$2.5x_1 + 1.5x_2 \le 1$$

$$4x_1 - x_2 \le 1$$

$$1.75x_1 + 2.5x_2 \le 0.5$$

$$1.5x_1 + 4x_2 \ge 0.5$$

$$x_1, x_2 \ge 0$$

$\min \overline{z_1}^{l} = -1.45$
$\min \overline{z_2}^{l} = 1.71$

An LP problem, which is equivalent to the MOLFP problem, is constructed according to the proposed algorithm as follows:

$$\min \overline{U^L} = (-4x_1 + 0.5x_2 + 0.5) + (1.45)(0.25x_1 + 0.5x_2 + 0.5) + (x_1 + 0.5x_2 + 0.5) - (1.71)(0.5x_1 + 0.5x_2 + 0.25)$$

$$= -3.5x_1 + 0.9x_2 + 0.93$$

s.t

$$2.5x_1 + 1.5x_2 \le 1$$

$$4x_1 - x_2 \le 1$$

$$1.75x_1 + 2.5x_2 \le 0.5$$

$$1.5x_1 + 4x_2 \ge 0.5$$

$$x_1, x_2 \ge 0$$

The resulting $\overline{x_{1opt}}^{l}, \overline{x_{2opt}}^{l} = (0.23, 0.038)$ and $\overline{z_{1opt}}^{l}, \overline{z_{2opt}}^{l} = (-0.7, 1.95)$

Step 7: Solve the problem $\underline{U^U}$ and obtain the upper bound $\left[\underline{z_1}^{su}, \underline{z_2}^{su}\right]$ (R4) by using Guzel's proposal.

$$\underline{z_1}^{u} = \min \frac{-x_1 + 3x_2 + 2}{x_1 + 2x_2 + 1}$$

$$\underline{z_2}^u = \min \frac{5x_1 + 2x_2 + 2}{2x_1 + 3x_2 + 1}$$

s.t

$$x_1 + 0.5x_2 \leq 4$$

$$2x_1 - 3x_2 \leq 5$$

$$0.5x_1 + x_2 \leq 3$$

$$0.5x_1 + 2x_2 \geq 2$$

$$x_1, x_2 \geq 0$$

$\min \underline{z_1}^u = -0.02$
$\min \underline{z_2}^u = 0.8$

An LP problem, which is equivalent to the MOLFP problem, is constructed according to the proposed algorithm as follows:

$$\min \underline{U}^U = (-x_1 + 3x_2 + 2) + (0.02)(x_1 + 2x_2 + 1) + (5x_1 + 2x_2 + 2) - (0.8)(2x_1 + 3x_2 + 1)$$

$$= 2.42x_1 + 2.64x_2 + 3.22$$

s.t

$$x_1 + 0.5x_2 \leq 4$$

$$2x_1 - 3x_2 \leq 5$$

$$0.5x_1 + x_2 \leq 3$$

$$0.5x_1 + 2x_2 \geq 2$$

$$x_1, x_2 \geq 0$$

the resulting $\underline{x_{1opt}}^u, \underline{x_{2opt}}^u = (0, 1)$ and $\underline{z_{1opt}}^u, \underline{z_{2opt}}^u = (1.67, 1)$

Step 8: Solve the problem $\underline{U}^L$ and obtain the lower bound $\left[\underline{z_1}^{sl}, \underline{z_2}^{sl}\right]$ (R3) by using Guzel's proposal.

$$\underline{z_1}^l = \min \frac{-3x_1 + x_2 + 1}{0.5x_1 + x_2 + 0.5}$$

$$\underline{z_2}^l = \min \frac{2x_1 + x_2 + 1}{x_1 + x_2 + 0.5}$$

s.t

$$2x_1 + x_2 \leq 2$$

$$x_1 - 2x_2 \leq 2$$

$$x_1 + 2x_2 \leq 1$$

$$x_1 + 3x_2 \geq 1$$

$$x_1, x_2 \geq 0$$

$\min \underline{z_1}^l = -1.17$
$\min \underline{z_2}^l = 1.5$

An LP problem, which is equivalent to the MOLFP problem, is constructed according to the proposed algorithm as follows:

$$\min \underline{U^L} = (-3x_1 + x_2 + 1) + (1.17)(0.5x_1 + x_2 + 0.5) + (2x_1 + x_2 + 1) - (1.5)(x_1 + x_2 + 0.5)$$

$$= -1.91x_1 + 1.67x_2 + 1.84$$

s.t

$$2x_1 + x_2 \leq 2$$

$$x_1 - 2x_2 \leq 2$$

$$x_1 + 2x_2 \leq 1$$

$$x_1 + 3x_2 \geq 1$$

$$x_1, x_2 \geq 0$$

the resulting $\underline{x_{1opt}}^l, \underline{x_{2opt}}^l = (0.727, 0.090)$ and $\underline{z_{1opt}}^l, \underline{z_{2opt}}^l = (-1.14, 1.93)$

The optimal values and the optimal solutions of MOLFP (R1: R4) are given in Table 1.

Table 1: Optimal values and optimal solutions of numerical example

Problem	R1	R2	R3	R4
Optimal values $(\boldsymbol{z_1}, \boldsymbol{z_2})$	$(-0.7, 1.95)$ $(\overline{z_1}^l, \overline{z_2}^l)$	$(1.7, 0.89)$ $(\overline{z_1}^u, \overline{z_2}^u)$	(-1.14, 1.93) $(\underline{z_1}^l, \underline{z_2}^l)$	$(1.67, 1)$ $(\underline{z_1}^u, \underline{z_2}^u)$
Optimal values with Single obj. $(\boldsymbol{U^l}, \boldsymbol{U^u})$	0.159 $\overline{U^l}$	19 $\overline{U^u}$	0.603 $\underline{U^l}$	5.86 $\underline{U^u}$
Optimal solutions $(\boldsymbol{x_1}, \boldsymbol{x_2})$	(0.23, 0.038) $(\overline{x_1}^l, \overline{x_2}^l)$	(0, 3.5) $(\overline{x_1}^u, \overline{x_2}^u)$	(0.727, 0.090) $(\underline{x_1}^l, \underline{x_2}^l)$	(0, 1) $(\underline{x_1}^u, \underline{x_2}^u)$

So, for problem (2):

- $[\underline{U^l}, \underline{U^u}] = [0.603, 5.86]$ is the surely optimal range,
- $[\overline{U^l}, \overline{U^u}] = [0.159, 19]$ is the possibly optimal range,

- $([\underline{U^l}, \underline{U^u}]\ [\overline{U^l}, \overline{U^u}]) = ([0.603, 5.86]\ [0.159, 19])$ is the rough optimal range,
- The solutions $(0.23, 0.038)^t$ *and* $(0, 3.5)^t$ are two of the rather satisfactory solutions,
- The solutions $(0.727, 0.090)^t$ *and* $(0, 1)^t$ are two of the completely satisfactory solutions.

7. Conclusion

In this paper, Basic concepts of rough intervals are studied. Rough intervals are valuable and novel tools to process the uncertainty in decision making problems with fractional programming. A new assumption of MOLFP problems is presented in which all of the coefficients. In order to solve these types of problems, we presented that each one of them can be transformed into two MOLFP problems with interval coefficients. A numerical example is given to illustrate our motivation for considering MOLFP problems with rough intervals. That algorithm as a methodology for this problem has solved it successfully after converted the MOLFP problem into a single objective linear programming problem. Studying more properties of the presented models can be a subject for further research.

References

[1] A. N. Dheyab, Finding the optimal solution for fractional linear programming problems with fuzzy numbers. *Journal of Kerbala University*, *10*(3) (2012), pp. 105-110.

[2] A. Charnes and W.W. Cooper, Programming with Linear Fractional Functionals, *Naval Research Logistics Quarterly* 9 (1962), pp. 181-186.

[3] B. Liu, Theory and Practice of Uncertain Programming, Physica Verlag, New York, (2002).

[4] B. Martos and V. Whinston, Hyperbolic programming. Naval Research Logistics (NRL), 11(2) (1964), pp. 135-155.

[5] E. Fathy, Building fuzzy approach with linearization technique for fully rough multi-objective multi-level linear fractional programming problem, Iranian Journal of Fuzzy Systems, Vol. 18, No. 2, (2021), pp. 139-157.

[6] E. S. ammar and M. muamer, On Solving Fuzzy Rough Linear Fractional Programming Problem, International Research Journal of Engineering and Technology, Vol. 03, Issue 04, (2016), pp. 2099-2120.

[7] E. S. ammar and M. muamer, Algorithm for Solving Multi Objective Linear Fractional Programming Problem with Fuzzy Rough Coefficients, Open Science Journal of Mathematics and Application, Vol. 04, Issue 01, (2016), pp. 1-8.

[8] E. A. Youness, Characterizing solutions of rough programming problems. *European Journal of Operational Research, vol.* 168(3) (2006), pp. 1019–1029.

[9] E. B. Bajalinov, Linear-fractional programming: theory, methods, applications and software. *Kluwer Academic Publishers*, *Boston*, (2003).

[10] F. A. Pramy, An Approach for Solving Fuzzy Multi-Objective Linear Fractional Programming Problems, International Journal of Mathematical, Engineering and Management Sciences, Vol. 3, No. 3, (2018), pp. 280–293.

[11] J. Nasiri and M. Mashinchi, Rough set and data analysis in decision tables, *Journal of Uncertain Systems, vol.* 3(3) (2009), pp. 232–240.

[12] J. Krysinski, Rough sets in the analysis of the structure-activity relationships of antifungal imidazolium compounds, *Journal of Pharmaceutical Sciences, vol.* 84(2) (1995), pp. 243–248.

[13] K. Swarup, Linear fractional functionals programming. *Operations Research*, *13*(6) (1965), pp. 1029-1036.

[14] M. S. Osman, K. R. Raslan, O. E. Emam and F. A. Farahat, Solving Multi-level Multi-objective Fractional Programming Problem with Rough Intervals in the Objective Functions, British Journal of Mathematics & Computer Science, vol. 21(2), (2017), pp. 1-17.

[15] M. S. Osman, E.F. Lashein, E.A. Youness and T.E.M. Atteya, Mathematical programming in rough environment *Optimization. vol.* 60(5) (2011), pp. 603–611.

[16] M. Rebolledo, Rough intervals-enhancing intervals for qualitative modeling of technical systems. *Artificial Intelligence, vol.* 170 (2006), pp. 667–685.

[17] M. Arabani and M.A.L. Nashaei, Application of Rough Set theory as a new approach to simplify dams location, *Scientia Iranica, vol.* 13(2) (2006), pp. 152–158.

[18] N. Guzel, A proposal to the solution of multiobjective linear fractional programming problem. *Abstract and Applied Analysis* (Vol. 2013). Hindawi Publishing Corporation, (2013).

[19] N. Guzel & M. Sivri, Proposal of a solution to multi objective linear fractional programming problem. *Sigma Journal of Engineering and Natural Sciences*, vol *2*, (2005), pp. 43–50.

[20] R. S. Porchelvi, L. Vasanthi, & R. I. Hepzibah,. On solving multi objective fractional linear programming problems. *International Journal of Current Research*, *6*(8) (2014), pp. 8095-8102.

[21] R. Moore, *Interval Analysis*, Prentice-Hall, Englewood-Cliffs, NJ, (1966).

[22] S. Jain,. Modeling of Gauss elimination technique for multi-objective linear programming problem. *Journal of Novel Applied Sciences* (2012), *1*, pp. 25-29.

[23] S. Jain,. Modeling of Gauss elimination technique for multi-objective fractional programming problem. *South Asian Journal of Mathematics*, *4*(3) (2014), pp. 148-153.

[24] W. Dinkelbach, On nonlinear fractional programming. *Management Science*, *13*(7) (1967), pp. 492-498.

[25] Y. Weiguo, L. Mingyu and L. Zhi, Variable precision rough set based decision tree classifier, *Journal of Intelligent and Fuzzy Systems, vol.* 23(2) (2012), pp. 61–70.