An Optimal Policy for Items with Linear Demand Rate, Variable Deterioration Rate Under Shortage and Lead-Time

[1]Bhawna Gupta, [2]Sangeeta Gupta,[3] Sweta Srivastav
[1,2,3]Sharda University, Greater Noida, India
Department of Mathematics
[2]sangeeta.gupta@sharda.ac.in , [3]sweta.srivastav@sharda.ac.in

Abstract

In this research we have developed a deterministic inventory model for anitem having linear demand in variable deterioration rate. Shortage is allowed and fully backlogged.In developing the model, we have assumed that lead time is not equal to zero. Here we developed an optimal policy to that minimize that the total average cost.

Model is illustrated by a suitable numerical example and sensitivity analysis has been carry-out.

Keywords: Inventory, variable rate of deterioration, Permissible delay in payment, fullybacklogging, having linear demand.
AMS SUBJECT CLASSIFICATION: 90B05.

I. INTRODUCTION

Inventory management and control are an important part of enterprise cost control.

In business scenarios, deterioration is inevitable. It has recently become more and more important. The effects of deterioration are very important in many inventory systems. Degradation is defined as modification, damage, decay, deterioration, obsolescence, loss of utility, or marginal loss of asset value, leading to reduce the practicality of the original. The largest items will deteriorate over time. Some items have a low rate of deterioration, such as hardware, glassware, toys, steel, etc.

No need to consider deterioration when determining the economic situation batch. Some items, such as food, medicine, vegetables, blood, gasoline, photographic film, medicines, chemicals, medicines, electronic components, radioactive substances, grains, such as wheat, potatoes, onions, fish, etc. Their shelf use is limited life expectancy deteriorates rapidly over time. Therefore, the impact of the deterioration of physical goods,it cannot be ignored in many inventory systems.

Constant demand rate hypothesis does not always apply to many inventory items, for example fashionable clothes, delicious food, cars with improved mechanisms, etc.

It has experienced fluctuations in demand rates. Many products have gone through an upswing period demands in the growth phase of the product life cycle, for example the arrival of new phones and laptops etc. on the market. On the other hand, the demand for certain products mayreduce due to the introductionof more attractive products that affect customers first choice. In addition, the age of the inventory has a negative impact on demand due to

Consumers lose confidence in quality due to physical loss of goods. Thisphenomenon prompted many researchers to develop deteriorating inventory models time-varying demand model.

Shah and Jaiswal (1997) presented an inventory model for deteriorated items with a constant deterioration rate. Aggarwal developed an inventory management model and modified and corrected the error in Shah Jaiswal's (1997) analysis and calculated the average cost of inventory.
Dave and Patel (1981) considered an inventory model for deteriorated items with demand proportional to time. Hollier and Mak (1983) developed with linear demand, Hariga and Benkherouf (1994) developed the model with exponential demand.
The study of spoilage is very important because of the different rates of spoilage of many products. Misra (1975) developed an economic order quantity (EOQ) model with a Weibull rate of deterioration that takes inflationary effects into account. Saurabh Srivastava and Harendra Singh (2017) developed a deterministic inventory model for items with linear demand, variable spoilage, and partial backlog. M. Maragatham and R. Palani (2017) formulatean inventory model for deteriorating items with price dependent demand and shortages.
In this paper we develop an inventory model for an item having linear demand in variable deterioration rate. Shortage is allowed and fully backlogged.In developing the model, we have assumed that lead time is not equal to zero.

II. ASSUMPTIONS AND NOTATIONS

The following notations are used to develop the model:

- c The unit cost per item.
- A The ordering cost of inventory/order
- A_D Amount of material deterioration during a cycle time.
- C_1 Holding cost per unit item per unit time.
- C_2 Shortage cost per unit item per unit time.
- t_1 Time at which shortage start.
- T Length of each ordering cycle.
- $\theta(t)$Time dependent deterioration cost
- $D(t)$ Demand rate
- LLead time
- P_CPurchasecost
- S_c Shortage cost
- C_D Total deterioration cost per cycle
- Q Maximum inventory level
- $I(t)$The inventory level at time t
- TCThe minimum average total cost per unit

Assumptions
The proposed inventory model is developed under the following assumptions;

- The inventory system involves only one item.
- The problem horizon is infinite.
- Replenishment occurs instantaneously at an infinite rate and the lead time is constant.
- The demandrate D(t) at any instant t is positive and is defined by

$$D(t) = \begin{cases} a + bt \ \ I(t) > 0 \\ D_0 I(t) \leq 0 \end{cases}$$

  where a, b and D_0all are constants such that a > 0, b > 0 and is initial demand.
- Shortages are allowed.
- The rate of deterioration at any time t > 0 follow the two parameter Weibull distribution as $\theta = \alpha\beta t^{(\beta-1)}$, where $\alpha(0 < \alpha < 1)$is the scale parameter and $\beta(> 0)$is the shape parameter.
- There is no repair or replenishment of the deteriorated items during the inventory cycle.
- The inventory is replenished only once in each cycle.
- During lead time shortages are allowed
- Ordering quantity is $Q+LD(p)$ when $t=L$.

III. MATHEMATICAL MODEL ANDSOLUTION

We consider the inventory deterioration model with linear demand and variable ratiodeterioration. The replenishment takes place at time t = 0, when the stock level reaches its level Q maximum from t = 0, due to demand and deterioration, stock levels decrease att_1. At time t_1, the stock level reaches zero, so the shortage begins at timet $= t_1$ tot $=$ T and all demand during the shortage period $[t_1, T]$is fully backward.

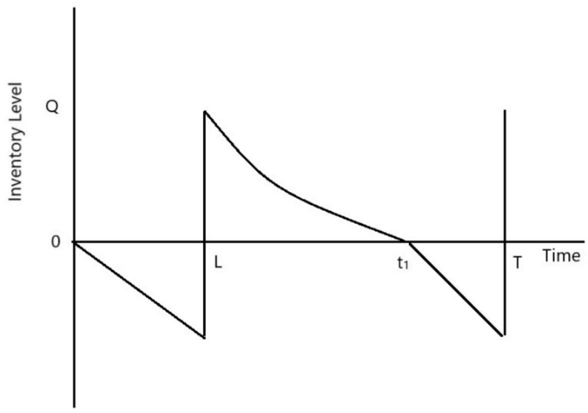


Fig. 1

Let $I(t)$be the inventory level at any time t,$(0 \leq t \leq t_1)$.The reduction of units in stockpile depend on demand and deterioration of units simultaneously. The differential equation that describes the instantaneous state over $[0, t_1]$ is given by:

$$\frac{dI(t)}{dt} + \alpha\beta t^{\beta-1} I(t) = D(t) \qquad L \leq t \leq t_1 \qquad \text{.............................(1)}$$

with initial condition $I(\mathrm{L}) = Q$ and the boundary condition$\mathrm{I}(\mathrm{t}_1) = 0$.

$\frac{dI(t)}{dt} = -D_0 \quad \mathrm{t}_1 \le \mathrm{t} \le \mathrm{T}$ ……………………..(2)

With the boundary conditions$\mathrm{I}(t_1) = 0, I(T) = -S_1$

From eqns(1)

$$I(t) = \left\{a(t_1 - t) + \frac{a\alpha}{(\beta+1)}\left(t_1^{\beta+1} - t^{\beta+1}\right) + \frac{b}{2}(t_1^2 - t^2) + \frac{\alpha b}{(\beta+2)}\left(t_1^{\beta+2} - t^{\beta+2}\right)\right\}\{1 - \alpha t^{\beta}\} \quad \mathrm{L} \le \mathrm{t} \le \mathrm{t}_1$$

…………………………..(3)

At time $\mathrm{t} = \mathrm{L}$, $\mathrm{I}\,(\mathrm{L}) = \mathrm{Q}$

$$Q = a(t_1 - L) + \frac{a\alpha}{(\beta+1)}\left(t1^{\beta+1} - L^{\beta+1}\right) + \frac{b}{2}(t_1^2 - L^2) + \frac{\alpha b}{(\beta+2)}\left(t_1^{\beta+2} - L^{\beta+2}\right\}\{1 - \alpha L^{\beta}\}$$

……………(4)

$Att = TS_1 = D_0(T - t_1)$ $, Since I(T) = -S_1$ ………………………(5)

The amount of material which deteriorate during one cycle is

$$A_D(t) = Q - \int_L^{t1} D(t)dt$$

$$A_D = \left\{a(t_1 - L) + \frac{a\alpha}{(\beta+1)}\left(t_1^{\beta+1} - L^{\beta+1}\right) + \frac{b}{2}(t_1^2 - L^2) + \frac{\alpha b}{(\beta+2)}\left(t_1^{\beta+2} - L^{\beta+2}\right)\right\}(1 - \alpha L^{\beta}) - \left\{\left(a(t_1 - L)\right) + \frac{b}{2}(t_1^2 - L^2)\right\}$$

……………………..(6)

The total variable cost will consist of the following costs

(a) The ordering cost of the materials, which is fixed per order for the present financial year.

(b) The deterioration cost is given by $c.\,AD$ which comes out to be

DETERIORTION COST=c. AD

$$c.A_D = \left\{a(t_1 - L) + \frac{a\alpha}{(\beta+1)}\left(t_1^{\beta+1} - L^{\beta+1}\right) + \frac{b}{2}(t_1^2 - L^2) + \frac{\alpha b}{(\beta+2)}\left(t_1^{\beta+2} - L^{\beta+2}\right)\right\}(1 - \alpha L^{\beta}) - \left\{\left(a(t_1 - L)\right) + \frac{b}{2}(t_1^2 - L^2)\right\}$$

………………………..(7)

Holding cost=$C_1 \int_L^{t1} I(t)dt$

$$H_C = C_1\Big[a(L^2 - Lt_1) + \frac{a\alpha}{(\beta+1)}\Big\{t_1^{\beta+1}(t_1 - L) - \frac{1}{(\beta+2)}\Big(t_1^{\beta+2} - L^{\beta+2}\Big)\Big\} + \frac{b}{2}\Big(t_1^2(t_1 - L) - \frac{1}{3}(t_1^3 - L^3)\Big) + \frac{\alpha b}{(\beta+2)}\Big(t_1^{\beta+2}(t_1 - L) - \frac{1}{(\beta+3)}\Big(t_1^{\beta+3} - L^{\beta+3}\Big)\Big) - a\alpha^2\Big(\frac{t_1}{\beta+1}\Big(t_1^{\beta+1} - L^{\beta+1}\Big) - \frac{1}{(2\beta+2)}\Big(t_1^{2\beta+2} - L^{2\beta+2}\Big)\Big) - \frac{b\alpha}{2}\Big(\frac{t_1^2}{(\beta+1)}\Big(t_1^{\beta+1} - L^{\beta+1}\Big) - \frac{1}{(\beta+3)}\Big(t_1^{\beta+3} - L^{\beta+3}\Big)\Big) - \frac{b\alpha^2}{(\beta+2)}\frac{t_1^{\beta+2}}{\beta+1}\Big(t_1^{\beta+1} - L^{(\beta+1)} - \frac{1}{2\beta+3}\Big(t_1^{2\beta+3} - L^{2\beta+3}\Big)\Big)\Big] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

Purchase cost

$$P_c = c(Q + LD_0)$$

$$= c\Big[\Big\{a(t_1 - L) + \frac{a\alpha}{(\beta+1}\Big(t_1^{\beta+1} - L^{\beta+1}\Big) - \frac{b}{(2)}(t_1^2 - L^2) + \frac{\alpha b}{(\beta+2)}\Big(t_1^{\beta+2} - L^{\beta+2}\Big)\Big\}\{1 - \alpha L^\beta\} + LD_0\Big] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(9)$$

Shortage cost

$$= C_2\left(-\int_{t1}^{T} I(t)dt\right)$$

$$= C_2\left(-\int_{t1}^{T} D_0(t_1 - t)dt\right)$$

$$= C_2\left(D_0\left(\frac{t_1^2}{2} + \frac{T^2}{2} - t_1T\right)\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(10)$$

Total variable cost function for one cycle is given by

$$TC = O_C + D_c + H_C + S_c + P_c$$

$$TC = A + c\Big[2\Big\{a(t_1 - L) + \frac{a\alpha}{(\beta+1)}\Big(t_1^{\beta+1} - L^{\beta+1}\Big) + \frac{b}{2}(t_1^2 - L^2) + \frac{\alpha b}{(\beta+2)}\Big(t_1^{\beta+2} - L^{\beta+2}\Big)\Big\}\big(1 - \alpha L^\beta\big) + LD_0 - \Big\{a(t_1 - L) + \frac{b}{2}(t_1^2 - L^2)\Big\}\Big] + C_1\Big[a(L^2 - Lt_1) + \frac{a\alpha}{(\beta+1)}\Big\{t_1^{\beta+1}(t_1 - L) - \frac{1}{(\beta+2)}\Big(t_1^{\beta+2} - L^{\beta+2}\Big)\Big\} + \frac{b}{2}\Big(t_1^2(t_1 - L) - \frac{1}{3}(t_1^3 - L^3)\Big) + \frac{\alpha b}{(\beta+2)}\Big(t_1^{\beta+2}(t_1 - L) - \frac{1}{(\beta+3)}\Big(t_1^{\beta+3} - L^{\beta+3}\Big)\Big) - a\alpha\Big(\frac{t_1}{(\beta+1)}\Big(t_1^{\beta+1} - L^{\beta+1}\Big)\Big) - \frac{a\alpha^2}{(\beta+1)}\Big(\frac{t_1^{\beta+1}}{(\beta+1)}\Big(t_1^{\beta+1} - L^{\beta+1}\Big) - \frac{1}{(2\beta+2)}\Big(t_1^{2\beta+2} - L^{2\beta+2}\Big)\Big) - \frac{b\alpha}{2}\Big(\frac{t_1^2}{(\beta+1)}\Big(t_1^{\beta+1} - L^{\beta+1}\Big) - \frac{1}{(\beta+3)}\Big(t_1^{\beta+3} - L^{\beta+3}\Big)\Big) - \frac{b\alpha^2}{(\beta+2)}\Big(\frac{t_1^{\beta+2}}{(\beta+1)}\Big(t_1^{\beta+1} - L^{\beta+1}\Big) - \frac{1}{2\beta+3}\Big(t_1^{2\beta+3} - L^{2\beta+3}\Big)\Big)\Big] + C_2\Big[D_0\Big(\frac{t_1^2 + T^2}{2} - t_1T\Big)\Big] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(11)$$

Our objective is to determine optimum value of Q to minimize TC. The values of t_1 for which

$\frac{dTC}{dt_1}$=0

$\frac{d^2TC}{dt_1^2} > 0$ satisfying the condition.

The optimal solution of the equation (11) is obtained by using MATLAB software. This has been illustrated by the following numerical example.

IV NUMERICAL PROBLEM

In this section, here we provide a numerical example for better understanding of the inventory model.

A = 200 , D =7 , a=5 , b=2 , $\alpha = 0.005$, $\beta = 0.4$, $L = 15 days$, $C_1 = 0.8$, $C_2 = 5$, $c = 2.2$, $T = 1 year$

Solving equation and adding data the optimal shortage period $t_1 \cong$ 211 *days and total cost* per unit time TC =210.11Rs

V SENSITIVITY ANALYSIS

Now, study the variation of the Lead time ; L , deterioration rate ; $\alpha(Scale parameter)$, $\beta(shape parameter)$, on decision variable and objective functions.

TABLE 1 (Effects of L and $'\alpha'$ *with* $\beta = 1.5$)

L/α	10 days	20 days	30days	40 days
0.02	$t_1 = 209$ TC=209.88	$t_1 = 211$ TC=209.92	$t_1 = 212$ TC=209.96	$t_1 = 213$ TC=210
0.04	$t_1 = 207.37$ TC=209.98	$t_1 = 208.78$ TC=209.98	$t_1 = 210.15$ TC=210.02	$t_1 = 211.57$ TC=210.06
0.06	$t_1 = 205.45$ TC=210	$t_1 = 206.85$ TC=210.03	$t_1 = 208.29$ TC=210.07	$t_1 = 209.75$ TC=210.114
0.08	$t_1 = 203.60$ TC=210.05	$t_1 = 205.52$ TC= 210.08	$t_1 = 206.49$ TC=210.12	$t_1 = 207.99$ TC=210.16

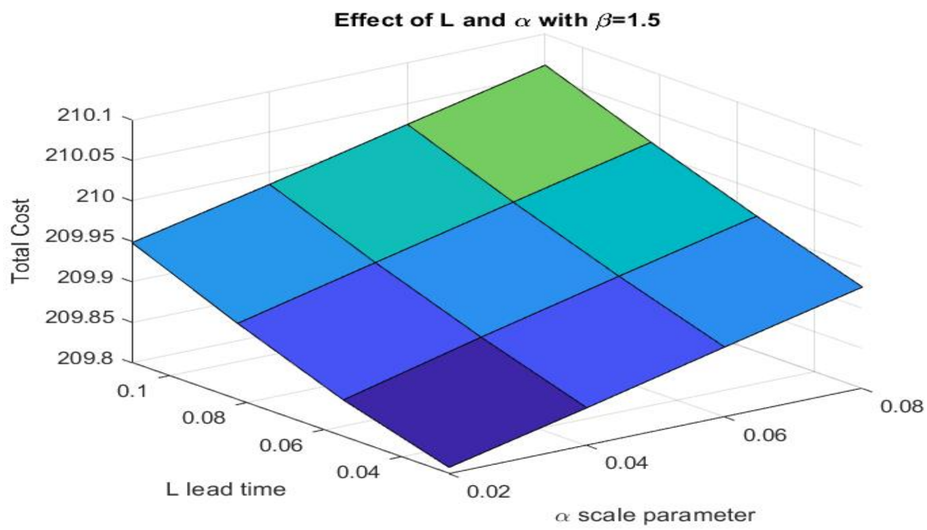


TABLE 2 (Effects of L and β $with\ \alpha = 0.02$)

L/$\boldsymbol{\beta}$	10days	20 days	30days	40days
1.5	$t_1 = 209.34$ TC=209.88	$t_1 = 210.70$ TC=209.92	$t_1 = 212.07$ TC=209.96	$t_1 = 213.45$ TC=210.012
1.7	$t_1 = 209.54$ TC=209.85	$t_1 = 210.88$ TC=209.89	$t_1 = 212.23$ TC=209.93	$t_1 = 213.60$ TC=209.98
1.9	$t_1 = 209.72$ TC=209.82	$t_1 = 211.05$ TC=209.86	$t_1 = 212.40$ TC=209.91	$t_1 = 213.75$ TC=209.96
2.1	$t_1 = 209.89$ TC=209.80	$t_1 = 211.21$ TC=209.84	$t_1 = 212.55$ TC=209.89	$t_1 = 213.89$ TC=209.94

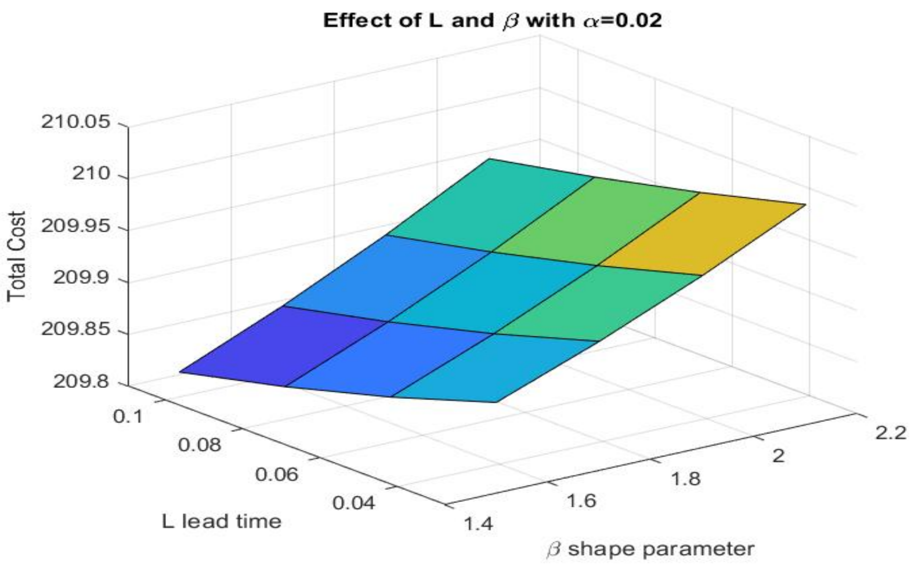
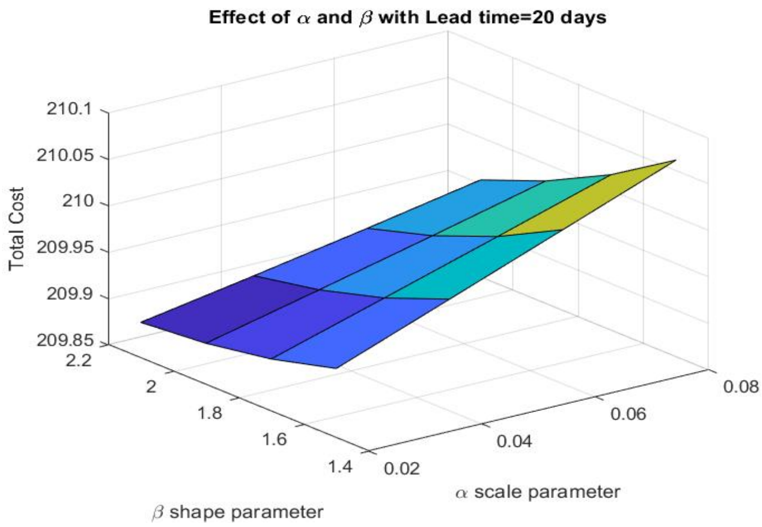


TABLE 3 (Effects of $\alpha\ and \beta\ \ with\ \ L = 20\ days$)

$\boldsymbol{\alpha/\beta}$	0.02	0.04	0.06	0.08
1.5	$t_1 = 210.70$ TC=209.92	$t_1 = 208.74$ TC=209.98	$t_1 = 206.85$ TC=210.03	$t_1 = 205.02$ TC=210.08
1.7	$t_1 = 210.88$ TC=209.89	$t_1 = 209.10$ TC=209.98	$t_1 = 207.38$ TC=209.99	$t_1 = 205.71$ TC=210.03
1.9	$t_1 = 211.05$ TC=209.86	$t_1 = 209.44$ TC=209.91	$t_1 = 207.88$ TC=209.95	$t_1 = 206.37$ TC=209.99
2.1	$t_1 = 211.21$ TC=209.84	$t_1 = 209.76$ TC=209.88	$t_1 = 208.34$ TC=209.91	$t_1 = 206.98$ TC=209.95



VI CONCLUSIONS:

The economic ordering policy suitable for the items having variable rate of deterioration, linear demand pattern and the fully backloggedin next replenishment along with a sensitivity analysis, which shows that:

- As the lead time (L) increase, t_1 (inventory finishing time) increase significantly and the total cost increase significantly.
- As the shape parameter (α) of deterioration rate increase, t_1 (inventory finishing time) decrease significantly and total inventory cost increase significantly.
- As the scale parameter (β) of degenerate rate increase, t_1 (inventory finishing time) increase marginally and total inventory cost decrease marginally.

REFERENCES

1. AGGARWAL, S. P. and JAGGI, C. K.: Ordering policies of deteriorating items under permissible delay in payments. Journal of the Operational Research Society, 46, 658 662,(1995).

2. CHAPMAN, C. B. and WARD, S. C., COOPER, D. F. and PAGE, M. J.: Credit policy and InventoryControl. Journal of the Operational Research Society, 35, 1055 1065, (1984).

3. CHUNG, K. J. and LIAO, J. J.: The optimal ordering policy in a DCF analysis for deteriorating items when trade the credit depends on the order quantity. International Journal of Production Economics, 100(1), 116 13

4. COVER, R.P., PHILIP, G.C.: An EOQ model for items with Weibull distribution deterioration. Am. Inst. Ind.Eng. Trans., 5, 323-326, (1973)

5. GANESH KUMAR, M.; UTHAYAKUMAR, R. "A two-echelon integrated inventory model under generalized leadTime distribution with variable backordering rate". International Journal of System Assurance Engineering and Management 10(4):552-562 (2019).

6. GHARE, P.M., SHADER, G.F.: A model for exponentially decaying inventories. J. Ind. Eng., 14, 238-243 (1963).

7. GOSWAMI, A., CHAUDHURI, K.S.: An EOQ model for deteriorating items with shortages and a linear trend in Demand. Journal ofoperational Research Society, 42, 1105-1110 (1991).

8. M. Maragatham and R. Palani, "An Inventory Model for Deteriorating Items with Lead Time price Dependent Demand and Shortages", Advances in Computational Sciences and Technology, 10(6), 1839-1847 (2017).

9. Sangeeta Gupta,Sweta Srivastav,Ragini, "An EOQ Model for Time Dependent Deteriorating Items with Residual Value and Fractionally Time-Varying Backlogging Under Permissible Delay in Payment", 9(6), 370-376(2020).

10. Saurabh Srivastava and Harendra Singh, "Deterministic inventory model for items with linear demand, variable deterioration and partial backlogging", Int. J. Inventory Research,4(4),333-349(2017).