Abstract: Pension fund needs to produce a high-income return to face actuarial expectations of different kinds of benefits. An asset allocation management model of a pension fund must consider a large planning horizon because of its long-term obligations. Asset allocation controls solvency of the fund by suitable investments and contribution policies to secure the pensioners future liabilities. Artificial intelligence approaches given by experts and accepted by decision-makers, provide a powerful tool for describing the uncertainty. A portfolio optimization model is introduced based on variance minimization at a required return level that secures the fund against insolvency risk. This method uses an Artificial Bee Colony Optimization Approach to the mean-variance defined by Markowitz so that future returns of the stocks are predicted where the ability of AI to improve predictive and prescriptive financial forecasting processes will change the world of finance management.

Keywords: Pension Fund; Mean and Variance; Artificial Bee Colony Optimization and Insolvency risk.

1. Introduction

Pension fund must be periodic evaluated by actuaries and predict annual cash flow of income and liabilities (outcomes). The sponsor of plan take decisions at certain time, as investment decision to decide which assets allocated to attain a return enough to pay the participants’ liabilities. The paper proposes model to assistance the decision maker to take this decision. Depending on earlier works of Markowitz [1], AI approaches which can help portfolio management teams analyze an investment operations and trading history, for example, to identify key drivers of performance and potential behavioral biases. With such realtime feedback, portfolio managers may be able to avoid suboptimal decisions and improve their results over time. The paper proposes modify Markowitz model by adding new constraint that responsible for secure the pension fund towards insolvency risk. One of the most important choices DB plan investment committees will make is deciding which asset classes they will invest in, and how much they will invest in each one. Few other choices will have a greater impact on plan sponsors achieving their goals, and thus it constitutes a crucial element of a well-designed investment policy statement. While some investment decisions, such as manager selection, can be outsourced, in most cases the decision on how to appropriately allocate assets rests with the investment committee.

Pension funds are becoming fundamental tools in financial markets. Nowadays, pension fund investments represent a considerable percentage of financial market operations. In a general perspective, there are two extremely different ways to manage a pension fund. First, the pension fund can be managed through Defined Benefit (DB) plans, where benefits are fixed in advance by the sponsor and contributions are
initially set and subsequently adjusted in order to maintain the fund in balance. In other words, DB plan provides a guarantee by the pension plan or government that a pension will be paid based on a certain formula in which contribution may not be tied actually to benefits. Secondly, pension fund can be managed through Defined Contribution (DC) plans, where contributions are fixed and benefits depend on the returns on fund portfolio. In other words, DC plan provides a pension plan in which a periodic contribution is prescribed and the benefit depends on the contribution plus the investment return.

Many papers used the types of pension plans as:

Boulier et al. [2], Vigna and Haberman [3], Deelstra et al. [4], Battocchio and Menoncin [5], call in DC pension funds, Haberman and Sung [6], Chang [7], Haberman et al. [8], Taylor [9], Chang et al. [10], Josa-Fombellida and Rincón-Zapatero [11] and Josa-Fombellida and Rincón-Zapatero [12], in DB pension funds, and Cairns [13], in both types of plans. Yufei et al. [14] they consider the portfolio optimization problem for a pension fund consisting of various government and corporate bonds. And aims to maximize the fund’s cash position at the end of the time horizon, while allowing for the possibility of bond defaults.

Portfolio optimization problem, which is sometimes referred to as portfolio selection problem, is a well-known problem in management, economy, and finance. Portfolio includes different financial securities, such as bonds and stocks owned by an organization or by individuals. One of the main issues when dealing with portfolio optimization is risk. Investors are always trying to balance between portfolio’s gains and risk. Thus, the goal is to select a portfolio with minimum risk at defined minimal expected returns. This further means reducing non-systematic risks to zero [15].

Markowitz specified the trade-off facing the investor: risk versus expected return. The investment decisions are not simply which securities to own, but how to divide the investor's wealth amongst them. This is a problem called “Portfolio Selection” hence the title of Markowitz’s seminal article published in the 1952 issue of the Journal of Finance. He identifies all feasible portfolios that minimize risk (as measured by variance or standard deviation or other measures) for a certain level of expected return and maximize expected return for a certain level of risk.

Portfolio optimization problem is a multi-criteria optimization problem where the goal is to minimize risks, while maximizing returns. Unfortunately, this problem approach has several shortcomings. First, this model is too simple for modeling real-world problem features. It does not capture all properties such as bounds of assets, transaction costs, cost of management, etc. Second, it can be quite difficult to gather enough data for risk and returns evaluation. Third, the estimation of return and covariance from historical data is very prone to measurement errors. Covariance matrix is used for defining the risk [16].

Portfolio optimization problem is being solved using different methods and techniques. Linear programming method, parametric quadratic programming technique and integer programming were successfully applied to solving vague portfolio selection problem.

In practice, market frictions, investor preferences, investment strategies, company policies of investment firms etc., have resulted in complex objectives and constraints that have made portfolio optimization problem more difficult, if not intractable. The complex mathematical models defining the portfolio have found little help from traditional or analytical techniques in their efforts to have optimal portfolios, forcing the need to look for non-traditional algorithms and non-orthodox approaches from the broad discipline of Computational Intelligence.

Fortunately, the emerging and fast-growing discipline of metaheuristics, a sub discipline of computational intelligence, has refreshingly become to be a solutions for all the ills of such of these famous problem models. Metaheuristics has not just turned out to be a viable alternative for solving intractable optimization problems, but in various cases has become to be the only alternative to solve the complex problems of concerned. Metaheuristic approaches represent efficient ways to deal with complex optimization problems and are applicable to both continuous and combinatorial optimization problems [17].

Nature-inspired Metaheuristics is a popular and active research area which relies on natural systems for the solution of optimization problems and is what has been applied to solve insolvency risk problem of pension fund by using portfolio optimization models which discussed in this thesis.
With the application of additional real-world constraints on the basic portfolio optimization formulation, the problem becomes harder for solving. In this case, traditional techniques and methods cannot generate satisfying results, and the use of heuristic and metaheuristic methods is more favorable. In some cases, problem characteristics, such as its size and constraints, or real world requirements, such as limited precision in estimating instance parameters or very limited computation time allowed, make traditional methods not particularly suitable for tackling large instances of the constrained portfolio selection problem, therefore researchers and practitioners have to resort to approximate algorithms and, in particular, to metaheuristics and hybrid techniques [18].

A metaheuristic is formally defined as an iterative generation process which guides a subordinate heuristic by combining intelligently different concepts for exploring and exploiting the search space. Thus, metaheuristics search for a perfect heuristic of a particular problem. Learning strategies are used to structure information in order to find efficiently near-optimal solutions. Key point in metaheuristics is that they do not guarantee to find the optimal solution, but the satisfying solution in a reasonable amount of execution time.

Fieldsend et al. [19] present the concept of use metaheuristics instead of the traditional quadratic programming approach to portfolio optimization because the difficulty of implement it when there are cardinality constraints (i.e. number of stocks in portfolio). Then recent approaches resolving this have used heuristic algorithms to search for points on the cardinality constrained frontier. However, these can be computationally expensive when the practitioner does not know a priori exactly how many assets they may desire in a portfolio, or what scale of return and risk they wish to be exposed to without recourse to analyzing the actual trade-off frontier.

Researchers paid a special attention on developing approximation methods such as heuristic and metaheuristic algorithms in literature about portfolio selection and pension funding, such as:

Evolutionary Algorithms (EA) and Swarm Intelligence (SI) approaches are two of the most preferred solution approaches for portfolio optimization. Metaxiotis & Liagkouras [20] presented a literature review of multi-objective EA while Kalayci et al. [21] presented a recent review of genetic algorithms for portfolio optimization.

Retirement savings plans putting individuals “in the driver’s seat” are proliferating around the globe. Many countries have already taken steps to create processes for participants that apply behavioral economics principles to balance personal engagement with automated decision making mechanisms. Artificial intelligence already prevalent in many facets of our daily lives can be an ideal vehicle to build upon that progress by helping to create a more personalized participant experience than ever before. Doing so can enable better and more dynamic financial decisions by more engaged participants, thereby decreasing retirement financial security. It can also empower plan fiduciaries to fulfill their duties more effectively and efficiently and to design plans that ultimately deliver better retirement outcomes for participants.

Josa-Fombellida and Rincón-Zapatero [22] study the optimal asset allocation problem of a DB pension plan that operates in a financial market composed of risky assets whose prices are constant elasticity variance processes.

In this article modifies Markowitz’s model by adding new constraint that responsible for secure the pension fund against insolvency risk i.e. ability for cover all participant’s liabilities along horizon and use ABC approach for tackling large instances of the constrained portfolio selection problem.

The paper is organized as follows. The second section shows important definitions that useful for the problem. The third section shows the proposed model of the problem. The fourth section shows the fundamentals of ABC algorithm. The fifth section presents the problem formulation and data set for the experiments and numerical example. Finally, the sixth section presents a conclusion.

2. Definitions
Definition 1. The fundamental of the portfolio variance

By generalized variance is to numbers $R_1, \ldots, R_n$ with constants $\lambda_1, \ldots, \lambda_n \in \mathbb{R}$, the definition can be obtained as:

$$\text{Var}(\lambda_1 R_1 + \lambda_2 R_2 + \cdots + \lambda_n R_n) =$$

$$\lambda_1^2 \text{Var}(R_1) + \lambda_2^2 \text{Var}(R_2) + \cdots + \lambda_n^2 \text{Var}(R_n) + 2\sum_{i \neq j} \lambda_i \lambda_j \text{Cov}(R_i, R_j)$$

**Definition 2. Markowitz’s model:**

$$\text{Min } \text{Var}(R_p)$$

Subject to

- $E(R_p) \geq K$
- $\sum_{i=1}^{n} x_i = 1$
- $x_i \geq 0$

Where

- $E(R_p)$: the expected portfolio return, $E(R_p) = \sum_{i=1}^{n} x_i E(R_i)$
- $\text{Var}(R_p)$: The variance of portfolio,
- $\text{Var}(R_p) = \sum_{i=1}^{n} x_i^2 \text{Var}(R_i) + 2\sum_{i \neq j} x_i x_j \text{Cov}(R_i, R_j)$
- $\text{Cov}(R_i, R_j)$: The covariance of returns $R_i$ and $R_j$
- $x_i$: is money allocated percentage at asset $i$
- $K$: certain return

3. Proposed model

Let the portfolio consisting of $n$ assets and operating over next time of one period, $M$ denotes the fund’s cash level at the start of the certain period, $R_i$ denote the return at the end of this period, $P$ denote the total pension payments plus all additive payments related the plan to be made during this period and $C$ denote the contribution to be made during this period obtained by actuaries. Hence we can write the proposed model as

$$\text{Min } \text{Var}(R_P) = \sum_{i=1}^{n} \text{Var}(R_i) x_i^2 + 2\sum_{i \neq j=1}^{n} \text{cov}(R_i; R_j) x_i x_j$$

Subject to

- $E(R_P) = \sum_{i=1}^{n} E(R_i) * x_i \geq K$
- $(1+ E(R_P)) \times (M) + C - P \geq 0$; (insolvency risk)
- $\sum_{i=1}^{n} x_i = 1$
- $\alpha_i < x_i < \beta_i; i=1,2,\ldots,n$
- $x_i \geq 0; i=1,2,\ldots,n$ (Short sell not allowed)

Where

$E(R_P)$: Expectation of return of portfolio
\( V(R_p) \): Variance of return (risk) of portfolio

\( K \): Required return satisfied the balance in pension plan and secures the fund against insolvency

\( cov(R_i, R_j) \): Covariance between returns of assets \( i,j \)

\( M \): Fund’s reserve of pension plan

\( C \): Contributions paid by scheme’s participants

\( P \): Grantees benefits paid by pension scheme plus all administrative expenses

\( x_i \): Proportion at assets \( i \)

\( \alpha_i \): Lower bound of asset \( i \)

\( \beta_i \): Upper bound of asset \( i \)

The adding constraint \((1+ E(R_p)))(M) + C - P \geq 0\) in detail:

This constraint meaning that all money in the fund \( M \) at start of period after investment optimization i.e. \((1+ E(R_p)))(M) plus yearly contributions \( C \) must be cover all liabilities plus all administrative expenses \( P \).

The optimized portfolio is found by minimizing the variance (risk) for a certain target return level, which determined by actuary mathematics rules, that reserve the pension fund from insolvency risk.

4. Constrained ABC algorithm

ABC is well-known population based swarm intelligence metaheuristic. It is inspired by the foraging behavior of bee swarms in nature. This approach firstly proposed by Karaboga [23], and developed by Karaboga and Basturk [24]. A serious difference between the ABC and other swarm intelligence algorithms is that in the ABC algorithm the possible solutions represent as food sources, not individuals (bees). In other algorithms, like PSO, each possible solution represents an individual of the swarm. In the ABC algorithm the quality of solution is represented as fitness of a food source. Fitness is calculated by using objective function of the problem.

In ABC metaheuristic, there are three types of artificial bees (agents):

i. Employed bees,

ii. Onlookers bees and

iii. Scouts bees.

Half of the colony is employed bees. The relation between employed bee and the food source is one-to-one, and that means that there is only one employed bee per each food source. If a food source becomes abandoned, employed bee that is mapped to that food source becomes a scout, and as soon as scout finds a new food source, it again becomes employed bee. In the ABC algorithm onlookers and employed bees carry out the exploitation procedures in the search space, while the scouts control the exploration procedures.

In the case of bees, the basic properties on which self-organization rely are as follows:

- Positive feedback: As the nectar amount of food sources increases, the number of onlookers visiting them increases.
- Negative feedback: The exploration procedures of a food source abandoned by bees are stopped.
- Fluctuations: The scouts carry out random search procedures for discovering new food sources.
- Multiple interactions: Bees share their information about food source positions with their nest mates on the dance area [15].

The framework of ABC is presented in Fig. 1, where there are three groups of bees constituting the whole colony employed, onlookers, and scouts bees. Firstly, employed bees start searching for food sources and swap the information they gather to onlooker bees. Then, onlookers make a decision and select some valuable food sources for further search. If the quality of the food source is not improved after a certain predetermined time, the employed bee will abandon it and turns into a scout. Then, the scout searches a new food source and the algorithm execute as mentioned next.
ABC algorithm, as an iterative algorithm, starts by associating each employed bee with randomly generated foodsource (solution). Each solution $x_i$ ($i = 1, 2, ..., SN$) is a $D$-dimensional vector, where $SN$ denotes the size of the population. Initial population of randomly generated solution is created using:

$$x_{i,j} = lb_j + rand(0,1) \cdot (ub_j - lb_j) \quad (1)$$

In each iteration, each employed bee detects a foodsource in its neighborhood, and evaluates its nectar amount (fitness). Discovery of a new, neighborhood solution is modeled with the following expression:

$$v_{i,j} = \begin{cases} x_{i,j} + \varnothing \cdot (x_{i,j} - x_{i,j}) ; feasible \\ x_{i,j} ; otherwise \end{cases} \quad (2)$$

Where $x_{i,j}$ is $j$-th parameter of the old solution $i$, $x_{k,j}$ is $j$-th parameter of a neighbor solution $k$, $\varnothing$ is a random number between -1 and 1.

Karaboga&Basturk[25] present a pseudo-code of the ABC metaheuristic for constrained optimization problems:

**Figure 1. The Flowchart of Artificial Bee Colony Algorithm**
1. Initialize the population
2. Evaluate the random population
3. Cycle=1
4. Repeat
5. Make new solutions for the employed bees by using Equation (2) and evaluate them
6. Apply selection process based on feasibility and fitness function.
7. Calculate the probability values \( p_i \) for the solutions \( x_i \), using fitness of the solutions
8. For each onlooker bee, produce a new solution \( v_{ij} \) by eq. (2) in the neighborhood of the solution selected depending on value of \( p_i \) and evaluate it
9. Apply selection process between \( v_i \) and \( x_i \) based greedy selection.
10. Determine the abandoned solutions by using the “limit” parameter for the scout, if they exist, replace them with new randomly produced solutions by eq. (1).
11. Memorize the best solution achieved so far.
12. Cycle = Cycle+1
13. Until cycle = MCN

We also note that in this case the fitness calculated using:

\[
fitness_i = \begin{cases} 
\frac{1}{1 + ObjFun_i}, & \text{if } ObjFun_i > 0 \\
1 + |ObjFun_i|, & \text{otherwise}
\end{cases}
\]

\( (3) \)

Where \( ObjFun_i \) is the value of the objective function which is the subject of optimization.

5. Problem formulation and data set for the experiments

This thesis used simple historical data set like in Zaher et al. [26]. The data encompasses historical return of a nine stocks portfolio of a period of nine years (2013-2021). Data set is shown in Table 1.

We can evaluate participants of some Egyptian funds at 1/1/2021 and its data and results as following:

- \( M \) (reserve of pension fund at start year) = 50000000 L. E.;
- \( K \) (the required rate of return secure the fund) = 0.09;
- \( P \) (all expected liabilities at first year) = 107330383;
- \( C \) (expected contribution at first year) = 47449742;

The actuary’s results:

<table>
<thead>
<tr>
<th>Years</th>
<th>Income</th>
<th>All liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>2021</td>
<td>47 449 742</td>
<td>107 330 383</td>
</tr>
</tbody>
</table>

Table 1 illustrate the promised benefits for the participants and their future contributions, from the evaluation’s results sponsor’s decisions should be invest the surplus to attain the required rate of return to satisfy fund’s balance.
In this part we can point to the case study Egyptian Stock Exchange (EGX). Table 2 shows these stocks. All data in this table consisting of the closing values of nine stocks from 1.2013 to 1.2021 are taken from the official web site of EgyptianStockExchange prices, sa.investing.com, on a yearly basis total of 9 observation periods.

### Table 2. Closed market value of the assets

<table>
<thead>
<tr>
<th>date</th>
<th>COMI</th>
<th>QNBA</th>
<th>VODE</th>
<th>OREG</th>
<th>EAST</th>
<th>ABUK</th>
<th>ETEL</th>
<th>GTHE</th>
<th>SWDY</th>
</tr>
</thead>
<tbody>
<tr>
<td>01/2013</td>
<td>15.11</td>
<td>7.51</td>
<td>86.9</td>
<td>15.93</td>
<td>2.31</td>
<td>8.4</td>
<td>14.5</td>
<td>4.42</td>
<td>2.05</td>
</tr>
<tr>
<td>01/2014</td>
<td>21.52</td>
<td>6.93</td>
<td>72.08</td>
<td>15.99</td>
<td>2.84</td>
<td>10.84</td>
<td>14.84</td>
<td>5.18</td>
<td>3.17</td>
</tr>
<tr>
<td>01/2015</td>
<td>35.33</td>
<td>8.97</td>
<td>80.06</td>
<td>19.25</td>
<td>4.89</td>
<td>10.68</td>
<td>12.21</td>
<td>4.65</td>
<td>5.07</td>
</tr>
<tr>
<td>01/2016</td>
<td>25.41</td>
<td>8.98</td>
<td>35.55</td>
<td>10.44</td>
<td>3.38</td>
<td>9.29</td>
<td>6.07</td>
<td>1.80</td>
<td>3.53</td>
</tr>
<tr>
<td>01/2017</td>
<td>61.15</td>
<td>13.85</td>
<td>66.77</td>
<td>10.61</td>
<td>8.83</td>
<td>6.36</td>
<td>12.30</td>
<td>7.08</td>
<td>8.40</td>
</tr>
<tr>
<td>01/2018</td>
<td>62.67</td>
<td>19.43</td>
<td>112.72</td>
<td>34.12</td>
<td>20.57</td>
<td>29.90</td>
<td>13.32</td>
<td>6.80</td>
<td>15.64</td>
</tr>
<tr>
<td>01/2019</td>
<td>66.42</td>
<td>20.21</td>
<td>108.06</td>
<td>13.01</td>
<td>16.86</td>
<td>23.30</td>
<td>13.99</td>
<td>4.52</td>
<td>17.68</td>
</tr>
<tr>
<td>01/2020</td>
<td>85.08</td>
<td>23.25</td>
<td>137.77</td>
<td>22.06</td>
<td>15.01</td>
<td>21.02</td>
<td>12.34</td>
<td>5.08</td>
<td>11.28</td>
</tr>
<tr>
<td>01/2021</td>
<td>63.34</td>
<td>17.31</td>
<td>130.00</td>
<td>18.56</td>
<td>14.80</td>
<td>23.45</td>
<td>11.66</td>
<td>5.08</td>
<td>10.17</td>
</tr>
</tbody>
</table>

### Table 3. The returns of the assets

<table>
<thead>
<tr>
<th>date</th>
<th>COMI</th>
<th>QNBA</th>
<th>VODE</th>
<th>OREG</th>
<th>EAST</th>
<th>ABUK</th>
<th>ETEL</th>
<th>GTHE</th>
<th>SWDY</th>
</tr>
</thead>
<tbody>
<tr>
<td>01/2014</td>
<td>6.41</td>
<td>-0.58</td>
<td>-14.82</td>
<td>0.06</td>
<td>0.53</td>
<td>2.44</td>
<td>0.34</td>
<td>0.76</td>
<td>1.12</td>
</tr>
<tr>
<td>01/2015</td>
<td>13.81</td>
<td>2.04</td>
<td>7.98</td>
<td>3.26</td>
<td>2.05</td>
<td>-0.16</td>
<td>-2.63</td>
<td>-0.53</td>
<td>1.90</td>
</tr>
<tr>
<td>01/2016</td>
<td>-9.92</td>
<td>0.01</td>
<td>-44.51</td>
<td>-8.81</td>
<td>-1.51</td>
<td>-1.39</td>
<td>-6.14</td>
<td>-2.85</td>
<td>-1.54</td>
</tr>
<tr>
<td>01/2017</td>
<td>35.74</td>
<td>4.87</td>
<td>31.22</td>
<td>0.17</td>
<td>5.45</td>
<td>-2.93</td>
<td>6.23</td>
<td>5.28</td>
<td>4.87</td>
</tr>
<tr>
<td>01/2018</td>
<td>1.52</td>
<td>5.58</td>
<td>45.95</td>
<td>23.51</td>
<td>11.74</td>
<td>23.54</td>
<td>1.02</td>
<td>-0.28</td>
<td>7.24</td>
</tr>
<tr>
<td>01/2019</td>
<td>3.75</td>
<td>0.78</td>
<td>-4.67</td>
<td>-21.11</td>
<td>-3.71</td>
<td>-6.60</td>
<td>0.67</td>
<td>-2.28</td>
<td>2.04</td>
</tr>
<tr>
<td>01/2020</td>
<td>18.66</td>
<td>3.04</td>
<td>29.72</td>
<td>9.05</td>
<td>-1.85</td>
<td>-2.28</td>
<td>-1.65</td>
<td>0.56</td>
<td>-6.40</td>
</tr>
<tr>
<td>01/2021</td>
<td>-21.74</td>
<td>-5.94</td>
<td>-7.77</td>
<td>-3.50</td>
<td>-0.21</td>
<td>2.43</td>
<td>-0.68</td>
<td>0.00</td>
<td>-1.11</td>
</tr>
</tbody>
</table>

Table 3 show the return’s values of these nine stocks by changes in their closed values as observations along 9 years.

### Table 4. The rate of return of assets

<table>
<thead>
<tr>
<th>date</th>
<th>COMI</th>
<th>QNBA</th>
<th>VODE</th>
<th>OREG</th>
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<th>ABUK</th>
<th>ETEL</th>
<th>GTHE</th>
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</tr>
</thead>
<tbody>
<tr>
<td>01/2014</td>
<td>0.42</td>
<td>-0.08</td>
<td>-0.17</td>
<td>0.00</td>
<td>0.23</td>
<td>0.29</td>
<td>0.02</td>
<td>0.17</td>
<td>0.55</td>
</tr>
<tr>
<td>01/2015</td>
<td>0.64</td>
<td>0.29</td>
<td>0.11</td>
<td>0.20</td>
<td>0.72</td>
<td>-0.01</td>
<td>-0.18</td>
<td>-0.10</td>
<td>0.60</td>
</tr>
<tr>
<td>01/2016</td>
<td>-0.28</td>
<td>0.00</td>
<td>-0.56</td>
<td>-0.46</td>
<td>-0.31</td>
<td>-0.13</td>
<td>-0.50</td>
<td>-0.61</td>
<td>-0.30</td>
</tr>
<tr>
<td>01/2017</td>
<td>1.41</td>
<td>0.54</td>
<td>0.88</td>
<td>0.02</td>
<td>1.61</td>
<td>-0.32</td>
<td>1.03</td>
<td>2.93</td>
<td>1.38</td>
</tr>
<tr>
<td>01/2018</td>
<td>0.02</td>
<td>0.40</td>
<td>0.69</td>
<td>2.22</td>
<td>1.33</td>
<td>3.70</td>
<td>0.08</td>
<td>-0.04</td>
<td>0.86</td>
</tr>
<tr>
<td>01/2019</td>
<td>0.06</td>
<td>0.04</td>
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<td>-0.22</td>
<td>0.05</td>
<td>-0.34</td>
<td>0.13</td>
</tr>
<tr>
<td>01/2020</td>
<td>0.28</td>
<td>0.15</td>
<td>0.28</td>
<td>0.70</td>
<td>-0.11</td>
<td>-0.10</td>
<td>-0.12</td>
<td>0.12</td>
<td>-0.36</td>
</tr>
<tr>
<td>01/2021</td>
<td>-0.26</td>
<td>-0.26</td>
<td>-0.06</td>
<td>-0.16</td>
<td>-0.01</td>
<td>0.12</td>
<td>-0.06</td>
<td>0.00</td>
<td>-0.10</td>
</tr>
</tbody>
</table>

Table 4 show the return’s rates of these nine stocks by dividing their values on closed market values of each stock as observations along 9 years.

The mean return on each asset and covariance matrix is given in Tables 5 and 6 respectively.
### Table 5. The mean of return of assets

<table>
<thead>
<tr>
<th>Asset</th>
<th>Mean of return</th>
</tr>
</thead>
<tbody>
<tr>
<td>COMI</td>
<td>0.29</td>
</tr>
<tr>
<td>QNBA</td>
<td>0.14</td>
</tr>
<tr>
<td>VODE</td>
<td>0.14</td>
</tr>
<tr>
<td>OREG</td>
<td>0.24</td>
</tr>
<tr>
<td>EAST</td>
<td>0.41</td>
</tr>
<tr>
<td>ABUK</td>
<td>0.42</td>
</tr>
<tr>
<td>ETEL</td>
<td>0.04</td>
</tr>
<tr>
<td>GTHE</td>
<td>0.27</td>
</tr>
<tr>
<td>SWDY</td>
<td>0.34</td>
</tr>
</tbody>
</table>

### Table 5. The covariance matrix of returns of assets

<table>
<thead>
<tr>
<th></th>
<th>COMI</th>
<th>QNBA</th>
<th>VODE</th>
<th>OREG</th>
<th>EAST</th>
<th>ABUK</th>
<th>ETEL</th>
<th>GTHE</th>
<th>SWDY</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>COMI</strong></td>
<td>0.305</td>
<td>0.103</td>
<td>0.167</td>
<td>-0.001</td>
<td>0.281</td>
<td>-0.176</td>
<td>0.196</td>
<td>0.529</td>
<td>0.258</td>
</tr>
<tr>
<td><strong>QNBA</strong></td>
<td>0.103</td>
<td>0.070</td>
<td>0.100</td>
<td>0.117</td>
<td>0.162</td>
<td>0.112</td>
<td>0.070</td>
<td>0.177</td>
<td>0.118</td>
</tr>
<tr>
<td><strong>VODE</strong></td>
<td>0.167</td>
<td>0.100</td>
<td>0.217</td>
<td>0.253</td>
<td>0.294</td>
<td>0.257</td>
<td>0.162</td>
<td>0.371</td>
<td>0.208</td>
</tr>
<tr>
<td><strong>OREG</strong></td>
<td>-0.001</td>
<td>0.117</td>
<td>0.253</td>
<td>0.800</td>
<td>0.360</td>
<td>1.073</td>
<td>0.030</td>
<td>0.001</td>
<td>0.175</td>
</tr>
<tr>
<td><strong>EAST</strong></td>
<td>0.281</td>
<td>0.162</td>
<td>0.294</td>
<td>0.360</td>
<td>0.534</td>
<td>0.456</td>
<td>0.238</td>
<td>0.572</td>
<td>0.416</td>
</tr>
<tr>
<td><strong>ABUK</strong></td>
<td>-0.176</td>
<td>0.112</td>
<td>0.257</td>
<td>1.073</td>
<td>0.456</td>
<td>1.798</td>
<td>-0.012</td>
<td>-0.253</td>
<td>0.256</td>
</tr>
<tr>
<td><strong>ETEL</strong></td>
<td>0.196</td>
<td>0.070</td>
<td>0.162</td>
<td>0.030</td>
<td>0.238</td>
<td>-0.012</td>
<td>0.193</td>
<td>0.460</td>
<td>0.213</td>
</tr>
<tr>
<td><strong>GTHE</strong></td>
<td>0.529</td>
<td>0.177</td>
<td>0.371</td>
<td>0.001</td>
<td>0.572</td>
<td>-0.253</td>
<td>0.460</td>
<td>1.225</td>
<td>0.487</td>
</tr>
<tr>
<td><strong>SWDY</strong></td>
<td>0.258</td>
<td>0.118</td>
<td>0.208</td>
<td>0.175</td>
<td>0.416</td>
<td>0.256</td>
<td>0.213</td>
<td>0.487</td>
<td>0.372</td>
</tr>
</tbody>
</table>

### 5.1. Problem formulation

The goal is to select weights of the each asset in the portfolio in order to minimize the portfolio’s risk at certain portfolio’s return. We add a certain constraint that control of solvency of pension fund problem with another constraints. The expected return of each individual security is presented as follows:

$$E(W_i) = w_i R_i$$  \hspace{1cm} (4)

Where $w_i$ denotes the weight of individual asset $i$, and $R_i$ is the expected return of $i$. Total expected return of the portfolio $P$ can be formulated as follows:

$$E(P) = \sum_{i=1}^{n} E(W_i)$$ \hspace{1cm} (5)

Where $n$ is the number of securities in the portfolio $P$. In our problem formulation, first goal is to satisfying portfolio’s expected return and the weight of securities are lies in their bounds and thus, the expression shown in (5) is greater than the required return that secure the fund towards insolvency risk where the fund’s risk should be minimized.

The objective function of the portfolio variance (risk) is presented as a polynomial of second degree which consider as the insolvency risk of pension fund can be present as:

$$\text{Min} \quad \text{Var}(P) = \sum_{i=1}^{n} \text{Var}(R_i) w_i^2 + 2 \sum_{i \neq j=1}^{n} \text{cov}(R_i, R_j) w_i w_j$$ \hspace{1cm} (6)

Where $\text{Var}(R_i)$ is variance of asset $i$, and $\text{cov}(R_i, R_j)$ is covariance between securities $i$ and $j$. 
5.2. ABC parameters setup

In this subsection, we present experimental results for testing ABC metaheuristics for pension fund's risk optimization problem. (See subsection 5.1 for problem formulation). All tests were performed on Intel Core i5 processor with 4GB of RAM memory, Windows 10 x64 operating system and Visual Studio 2021 with .NET 4.5 Framework.

Solution number SN was set to 50, and maximum cycle number MCN was set to 1000, yielding a total of 50,000 objective function evaluations (50*1000).

Limit parameter is calculated using:

\[
\text{Limit} = \frac{\text{MCN}}{\text{SN}}
\]  

(7)

Thus, in this case, limit is set to 20 (1000/50). According to ABC experimental studies, limit calculated with (7) establishes optimal balance between exploitation and exploration [23].

Since the portfolio of pension fund's reserve contains nine stocks, dimension D of a problem is 9. Each food source in the population is a 9-dimensional vector. In initialization phase, food source x is generated using the following expression:

\[
x_i = w_i^{low} + \text{rand}(0,1) \times (w_i^{up} - w_i^{low})
\]

(8)

Where rand(0, 1) is a random number uniformly distributed between 0 and 1.

5.3. Experimental results

By writing model's code and having results.

The optimal allocation for the reserve of pension fund to secure the fund and reserve the balance for the pension's plan for the nine assets is:

Table 6. The result

<table>
<thead>
<tr>
<th>Assets</th>
<th>COMI</th>
<th>QNBA</th>
<th>VODE</th>
<th>OREG</th>
<th>EAST</th>
<th>ABEK</th>
<th>ETEL</th>
<th>GTHE</th>
<th>SWDY</th>
</tr>
</thead>
<tbody>
<tr>
<td>weights</td>
<td>0.4</td>
<td>0.4</td>
<td>0</td>
<td>0.11</td>
<td>0.01</td>
<td>0.06</td>
<td>0.02</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 6 illustrates the weights that satisfy the constraint and the objective function has minimum value. Then sponsor’s objectives are occur where risk (variance) is minimized which is objective function and equal 0.14.

6. Conclusion

In this paper, ABC algorithm for pension fund constrained portfolio optimization problem was presented. The implementation of the ABC for this problem was not found in the literature. The algorithm was tested on a set of nine stocks portfolio.

The results of the investigation reported in this paper show that the ABC swarm intelligence metaheuristic has potential for solving this problem.

ABC was applied only to the basic portfolio optimization problem definition. There is a large potential for applying metaheuristic techniques to this class of problems as pension fund portfolio problems, because they appear not to be investigated enough. In the subsequent work, original, as well as the version of the ABC will be applied to the extended-mean variance after adding the constraint which control of the insolvency risk of pension fund, and other pension funds models. Also, other swarm intelligence metaheuristics will be applied to various pension fund problem models and definitions.

We have analyzed the management of a pension funding process of a DB pension plan when the short interest rate is the yearly model. Yearly insolvency risk may be solved analytically when the benefits process is determined under a suitable selection of the technical interest rate and actuary determine the required return rate that does not expose pension fund to insolvency risk.

The components of the optimal portfolio investments in risky and riskless assets are the sum of all terms, and face the actuarial liability, depending on parameters of the randomness of benefits, all expenses and contributions where interest rate determined by actuaries.
We have done a case study of the pension fund and have all required data to show some properties of the model. The decision maker would check the fund’s finance status every short certain period. A portfolio optimization method which minimizes the variance of the portfolio is introduced and this method is applied to the 9 well known stocks of Egypt Stock Market. With its background this method has some serious advantages compared to the classical MV optimization which is introduced by Markowitz 69 years ago. Firstly, the required return for balance of pension’s fund can check short certain period and can reallocate if the required not satisfied. Secondly, the portfolio managers can add their subjective opinions to the model with the help of change of the parameters of ABC algorithm. Thirdly, this method does not require the limitations of classical MV optimization which are listed in detail of the classical MV optimization.

References