An Approach for Solving the Fixed Charge Transportation Problems

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ABSTRACT

This paper presents modified Vogel's method that solves the fixed charge transportation problems,the relaxed transportation problem proposed by Balinski in 1961 to find approximate solution for the fixed charge transportation problem (FCTP). This approximate solution is considered as a lower limit for the optimal solution of FCTP.This paperdeveloped the modified Vogel's methodto find an approximate solution used as a lower limit for the FCTP. This is better than Balinski's method in 1961. Myapproach relies on applying the Vogel's approximation method on the relaxed transportation problem. In addition, an illustrative numerical example is used to prove my hypothesis.

Key words: Transportation Problem, Fixed Charge, Vogel's Method.

1. Introduction

In a transportation problem (TP), products have to be transported from a number of sources to a number of destinations. Decisions have to be taken according to the amount of products transported between each two locations to minimize total transportation cost [1]. Typically, only a variable cost proportional to the number of products transported is afforded. However, in many real-world problems, a fixed/setup cost is also afforded when the transportation amount is positive [2]. This problem is referred to as fixed charge transportation problem (FCTP).This method starts with a linear formulation. Among these methods, branch-and-bound methods are one of the most effectiveones and had many applications. Ingeneral,thefixed charge problems can be properly stated as:

$$\text{Min } Z = c'x + d'y \quad (1)$$

$$Ax = b \quad (2)$$

$$x = 0 \quad (3)$$

$$y_j = \begin{cases} 0, & x_j = 0 \\ 1, & x_j > 0 \end{cases} \quad (4)$$

Where c, x, d, yare n-vectors, b is an m-vector and A is an $(m$ x n) matrix, It is well knownthat the optimal solution to equations (1 to 4) lies at an extreme point of the convex set S $=\{ x \mid Ax = b, x \geq 0 \}$, Hence solution methods may confine their search to the extreme points of the set S [3].

We present amodified Vogel's method. We show The fixed charge problems in Section 3, and the fixed charge transportation problem in Section 4. We present numerical examplein Section 5We conclude the paper in Section 6.

2. A Modified Vogel's Method

Step 1 Deduct the smallest entry from each of the elements of each row of the TT and place them on the right-top of corresponding elements.

Step 2 Apply the same operation on each of the columns and place them on the right-bottom of the corresponding elements.

Step 3 Place the Average Row Penalty (ARP) and the Average Column Penalty (ACP) just after and below the supply and demand amount consecutively within first brackets, which are the averages of the right top elements of each row and the right-bottom elements of each column consecutively of the TT.

Step 4 Identify the highest element among the ARPs and ACPs. If there are two highest elements or more, choose the highest element together with the smallest cost presented. If there are two smallest elements or more, choose any one of them despotically.

Step 5 Allocate x_{ij} = min (y_i, y_j) on the left top of the smallest entry in the (i, j) th of the TT.

Step 6 If $y_i < y_j$, leave the i^{th} row and readjust y_j as $y_j^* = y_j - y_i$
If i j a $> y_j$, leave the j^{th} column and readjust y_i as $y_i^* = y_i - y_j$
If $y_i = y_j$, leave either ith row or j-th column but not both.

Step 7 Repeat Steps 1 to 6 until the rim requirement satisfied.

Step 8

Calculate $z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$

, z being the minimum transportation cost and ij c are the cost elements of the TT

3. The fixed charge problems

Problems with fixed charges, such as knapsack (KP) and fixed charge linear programming problems (FCP), including the transportation type formulation (FCTP), have been historically treated as pure integer or mixed integer problems. The optimization literature presents several different methods for solving those problems. Those methods have different hypothesises about the effectiveness of their procedures. Whereas the problems with fixed charges are usually NP-hard,the computational time to get exact solution increases in a polynomial fashion and becomes extremely difficult and long very quickly as the size of the problem increases [4].

4. The fixedcharge transportation problem

Assume that there are m (i = 1, 2, . . ., m) suppliers and n (j = 1, 2, . . .,n)customers in a transportation problem. Supply i is ai units at eachsupplier, and demand j is b_j unitsat each customer. Let x_{ij} be the number of units shipped by supplier i to customer j. The cost of shipping x_{ij} units is composed of two parts: a variable cost per unit c_{ij}, and a fixed cost f_{ij} for opening this route. The objectiveis to minimize the total cost of meeting all demands, give all supply constraints [5,6].The fixed charge transportation problem (FCTP) is formulated as follows:

Minimize $Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$ (5)

$$y_{ij} = \begin{cases} 1 & if \quad x_{ij} > 0 \\ 0 & if \quad x_{ij} \leq 0 \end{cases}$$

$$\text{s.t.} \sum_{j=1}^{n} x_{ij} \geq a_i \qquad j = 1, 2, \ldots\ldots, n \qquad (6)$$

$$\sum_{i=1}^{m} x_{ij} \leq b_j \qquad i = 1, 2, \ldots\ldots, m \qquad (7)$$

$$\forall\ i,j \quad x_{ij} \geq 0$$

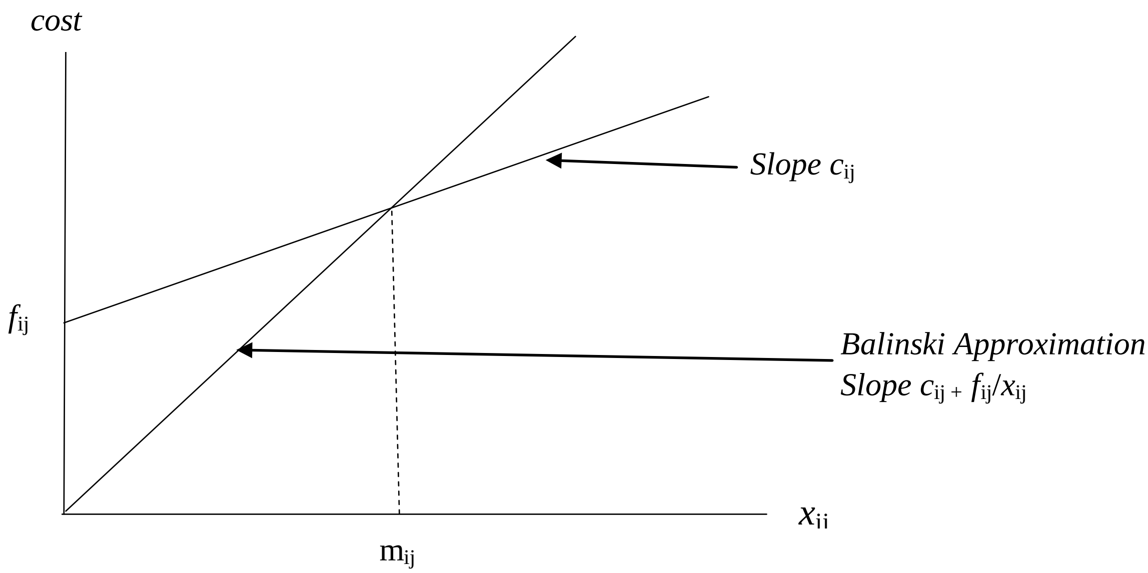

Fig.1: Balinski linear Approximation [8]

Balinski relaxed the integerconstraint on y_{ij}, with the property that under this relaxation, an FCTP problem is transformed into aclassical TP without fixed charge, where the unit transportationcosts [7]

$$C_{ij} = c_{ij} + f_{ij}/m_{ij}. \qquad (8)$$

$$\text{Where}\, m_{ij} = \begin{cases} \min(a_i, b_j) & if \;\; a_i, b_j > 0 \\ a_i & if \;\; b_j = 0 \\ b_j & if \;\; a_i = 0 \end{cases}$$

5. Numerical example

Consider a company with three factories in locations S_1, S_2, and S_3 which produces a specific type of product. There are three other locations D_1, D_2, and D_3 that receive this product as consumers. The supply Si, the demand D_j, the cost f_{ij} for opening the route (i, j) and the unit cost c_{ij} for transporting one unit of the given product from the source to the destination are given in table 1.

Table1: Cost matrix f_{ij}, c_{ij}[7]

	D_1	**D_2**	**D_3**	**Supply**
S_1	(20,1)	(10, 2)	(10, 3)	**15**
S_2	(20, 1)	(20, 2)	(20, 1)	**30**
S_3	(10, 3)	(20, 1)	(10, 2)	**15**
Demand	**20**	**20**	**20**	**60**

Step 1: The Balinski RTP can be represented as in table 2.

Table2: Balinski's RTP matrix with $C_{ij} = c_{ij} + f_{ij} / m_{ij}$

	D_1	**D_2**	**D_3**	**Supply**
S_1	2.33	2.67	3.67	**15**
S_2	2	3	2	**30**
S_3	3.67	2.33	2.67	**15**
Demand	**20**	**20**	**20**	**60**

Step 2: Calculation of the row differences and column differences as table 3 are

Table 3: row differences and column differences

	D_1	D_2	D_3	Supply
S_1	$2.33^{0}_{0.33}$	$2.67^{0.34}_{0.34}$	$3.67^{1.34}_{1.67}$	15
S_2	2^{0}_{0}	$3^{1}_{0.67}$	2^{0}_{0}	30
S_3	$3.67^{1.34}_{1.67}$	2.33^{0}_{0}	$2.67^{0.34}_{0.67}$	15
Demand	**20**	**20**	**20**	**60**

Step 3: The allocations with the help of ARP and ACP are

Table 4: ARP and ACP

	D_1	D_2	D_3	Supply	ARP				
S_1	2.33^{10}	$2,67^{5}$	3.67	15	0.56	0.17	0.17	0.34	----
S_2	2^{10}	3	2^{20}	30	0.33	0.5	----	----	----
S_3	3.67	2.33^{15}	2.67	15	0.56	0.67	0.67	----	----
Demand	35	30	25	60					
ACP	0.67	0.34	0.78						
	0.67	0.34	----						
	1	0.17	----						
	----	0.17	----						
	----	----	----						

Table 5: Final allocation

	D_1	D_2	D_3	Supply
S_1	10	5		**15**
S_2	10		20	**30**
S_3		15		**15**
Demand	**20**	**20**	**20**	**115**

The total variable cost of final distribution is 90, the total fixed cost is 55 and the total cost is 145. While the proposed algorithm finds an initial appropriate solution with total cost equals **145**.

Table 6: Total cost of our approach

Total cost $		
Balinski's[7]	Kowalski et al. [8]	Myapproach
155 $	153.33 $	**145 $**

6. Conclusion

This paper presents a modified Vogel's method for finding a solution to be used as a lower bound for the optimal solution of FCTP. This modified method has been compared to a set of problems. The RTP value which uses this algorithm can be considered as a better lower bound to the optimal solution of FCTP compared to the RTP value obtained by Baliniski's and simple branching algorithm for solving small scale fixed-charge transportation problem.

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References

[1] Kowalski K., Lev B., New approach to fixed charges problems (FCP). International Journal of Management Science and Engineering Management,(**2007**), 2:75-8.

[2] Sadagopan S., Ravindran A., A Vertex Ranking Algorithm for the Fixed Charge Transportation Problem. Journal of Optimization Theory and Applications,(**1982**), 2:221–230.

[3] N. Balakrishnan, 'Modified Vogel's Approximation Method for Unbalance Transportation Problem,' Applied Mathematics Letters 3(2), (**1990)**, 9,11.

[4] H.H. Shore, 'The Transportation Problem and the Vogel's Approximation Method', Decision Science 1(3-4), (**1970**), 441-457.

[5] V.J. Sudhakar, N. Arunnsankar, T. Karpagam, 'A new approach for find an Optimal Solution for Trasportation Problems', European Journal of Scientific Research 68,(**2012**), 254-257.

[6] Costa A., A survey on benders decomposition applied to fixed charge network design problems. Computers & Operations Research, 32:1, (**2005**), 429–1450.

[7] Balinski, M. Fixed cost transportation problems. Naval Research Logistics Quarterly, 8, (**1961**),41–54.

[8] Kowalski, k., Lev, B., Shen, W., Tu, W. A fast and simple branching algorithm for solving small scale fixed-charge transportation problem, Operations Research Perspectives, 1, (**2014**), 1–5.