FIXED POINT THEORY IN COMLEX VALUED METRIC SPACE

Abhishek Koundal

Department of Mathematics
Email ID:akmonu98@gmail.com
Chandigarh University, Mohali, Punjab

Abstract: The aim of this paper is to establish and prove several results on common fixed point for a pair of mappings satisfying more general contraction conditions portrayed by rational expressions having point-dependentcontrolfunctionsascoefficientsincomplexvaluedmetricspaces.Fixedpointtheory in complex valued metric space using contractive conditions, rational inequality, common limit range property for two pairs of mapping deriving common fixed-point results under a generalized altering distance functions, E.A and CLR property.Obtaining consecutive approximations to the fixed point of an approximate mapping is the goal of a variety of processes in numerical analysis and approximation theory. Our goal in this paper is to examine fixed point theory and its applications in metric spaces, as well as to develop several fixed-point theorems in entire metric spaces that generalize many renowned mathematicians' achievements.

Keywords: Fixed-point theory, Metric space, Complete Metric space, Continuous function.

INTRODUCTION

Azam et al. [1] introduced the concept of complex valued metric space by establishing a fixed-point result for mapping in complex valued metric space that fulfils a reasonable inequality. Several works have been published since then that deal with the fixed point hypothesis in complex valued metric space (see [3-11]and references in that) Fixed-point was first concentrated by Rao et al. [12].
findings for various mappings that satisfy a rational requirement in complex valued b-metric spaces (see [13-16] and the relevant references in. Sintunavaratel al. (9,10), Sitthikul and Saejung [11], and Singhetal.[8] have recently established fundamental fixed-point results by substituting the consistency of contractive condition to control functions in complex valued metric spaces. Inacontinuationof**[8,11,15,17],**somenormalfixed-point result for a couple of mapping fulfilling more broad contractive conditions including rational expression having point-subordinate control function as coefficients in complex valued b-metric spaces have been proved by manyauthors.

The well-known Banach contraction principal states that "If X is complete metric space and f is a contractionmappingonXintoitself,thenfhasuniquefixedpointinX".Manymathematiciansworked on this principal. Kannan proved that If T is self-mapping of a complete metric space X into itself satisfying:

$d(Tx,Ty)\leq[d(Tx,x)+d(Ty,y)]$ for all $x,y\in X$, where $\alpha\in\left(0,\frac{1}{2}\right)$.

Then T has unique fixed point in x.

Fisher, proved the result with

$d(Tx,Ty)\leq[d(Ty,x)+d(Tx,y)]$ for all $x,y\in X$, where $\alpha\in\left(0,\frac{1}{2}\right)$.

A similar conclusion was also obtained by Chatterjee

In 1977, the mathematician Jaggi, introduced the rational expression first time as:

$$d(Tx,Ty)\leq \alpha\alpha\alpha\alpha(xx,yy)+\beta\beta\frac{dd(xx,TTxx)dd(yy,TTyy)}{dd(xx,yy)} ffffeeaaaa \qquad xx,\ yy\in XX,\ xx\neq yy,\ 0\leq \alpha\alpha+\beta\beta<1.$$

In 1980 the mathematician Jaggi and Das obtained some fixed-point theorems with the mapping satisfying:

$$d(Tx,Ty)\leq \alpha\alpha\alpha\alpha(xx,yy)+\beta\beta\frac{dd(xx,TTxx)dd(yy,TTyy)}{dd(xx,yy)+dd(xx,TTyy)+dd(yy,TTxx)} aaffffeea\ aaa \quad xx,\ yy\in XX,\ \alpha\alpha+\beta\beta<1.$$

In the present paper we also finding a new rational expression, using complete metric spaces, which satisfy the many result of great mathematicians.

Preliminaries

Banach fixed point theorem [1] in complete metric space has been summed up in numerous spaces. In2011Azametal.[2]presentedthethoughtofcomplex–valuedmetricspaceandbuilt-upsufficient condition for the presence of common fixed point of a pair of mappings fulfilling a contractive condition. The possibility of complex-valued metric spaces can be abused to define complex-valued normed spaces and complex-valued Hilbert spaces, moreover it offers various research exercise in numerical examination. The theorems demonstrated by Azam et al. [2] and Bhatt et al. [18] utilizes the rational in equality in a complex valued metric space as contractive condition. In this paper. We presenttheideaofproperty(E.A)incomplex-valuedmetricspace,todemonstratesomenormalfixed-pointresultforafourfoldofself-mappingfulfillingacontractiveconditionof'max'type.Ouroutcomes sum up different theorems of customary metricspaces.

A real valued function from a set X X into R is an ordinary metric d, where X is a non-empty set. d:X X R is the code. A complex number zC is an ordered pair of real numbers with Re(z) as the first co-ordinate and Im as the second co-ordinate (z). As a result, a complex valued metric d is a function from a set XX into C, where X is a nonempty set and C is the complex number set.

That is, d: X×X →C.

let $z_{1,}z2\in C$, define a partial order on C as follows:

$z1\leq z2$if and only if $Re(z_1)\leq Re(z_2), Im(z_1)\leq Im(z_2)$

Itfollowsthat$z_1\leq z_2$ifoneofthefollowingconditionsissatisfied:

$Re(z_1)=Re(z_2), Im(z_1)<Im(z_2)$

$Re(z_1)<Re(z_2), Im(z_1)=Im(z_2)$

$Re(z_1)<Re(z2), Im(z_1)<Im(z_2)$

$Re(z_1)=Re(z2), Im(z_1)=Im(z_2)$.

In(1),(2)and (3), we have $|z_1|<|z_2|$.In (4), we have $|z_1|=|z_2|$.In particular,$z_1< z_2$ if $z_1\neq z_2$ and one of (1),(2),(3) is satisfy. In this case $|z_1|<|z_2|$.We will write $z_1< z_2$ if only (3) satisfy. Further,

$0\leq z_1<z_2\rightarrow|z_1|<|z_2|$

$z_1 \leq z_2$ and $z_2 < z_1 \rightarrow z_3$

Azam et al. [2] defined the complex valued metric space (X,d) in the following ways:

Defination1.1 Let x be a non-empty set. Suppose that the mapping d:X×X→C satisfies the following conditions:

(C1) 0≤d(x,y) for all x,y X and d(x,y)=0 if and only if x=y;

(C2) d(x,y)=d(y,x) for all x,yεX;

(C3) d(x,y)≤d(x,z)+d(z,y) for all $x,y,z^{\varepsilon}\varepsilon X$

Then d is called a complex valued metric on X, and (X,d) is called a complex valued metric space.

Common Fixed point Theorems Using Property (E.A) in complex valued Metric

spaces. Fixed Point Theorem (E.A) Property [19]

In this paper author proved some important fixed-point theorems using (E.A) property and (CLR) property in complex valued metric space in which the author also used the notion of partial order.

Theorem[a] Let (X,d) be a complex valued metric space and A, B, S, T: X→X be four self-mapping satisfying.

(i). A(X)⊆T(X), B(X⊆)S(X),

(ii). d(Ax, By) <k max (d (Sx, Ty), d (By, Sx), d (By, Ty),∀x,

y∈X,0<k<1, (iii). The pairs (A, S) and (B, T) are

weaklycompatible,

(iv). One of the pair (A, S) and (B, T) satisfy(E.A)

If the range of one of the mapping S(X) or T(X) ids complete subspace of Then mappings A, B, S and T have unique common fixed point in X.

Fixed Point Theorem Using (CLR)-Property

The notion of (CLR)-property was defined by Sintunavarat and Kumam [20] in a metric space for a pair of self-mapping, which have the common limit in the range of one of the mappings.

Definition: (The (CLR)-property [20]) Suppose that (X, d) is a metric space and f, g; X→X. Two mappingfandgaresaidtosatisfythecommonlimitintherangeofgpropertyif$\mathit{llllll}_{nn\rightarrow\infty} fx_n = \mathit{llllll}_{nn\rightarrow\infty} gx_n = gx$,
for some
x∈X.

InthecomplexvaluedmetricspacethedefinitionwillbesamebutthespaceXwillbeacomplexvalued metricspace.

Theorem[b]. Let (X, d) be a complex valued metric space and A, B, S, T; X→X be a four self-mapping satisfying:

(i). A(X)⊆T(X),

(ii). d (Ax, By)≤k max (d (Sx, Ty), d (By, Ty)) ∀x,

y∈X,0<k<1, (iii). The pairs (A, S) and (B, T) are

weaklycompatible.

If the pair (A, S) satisfy (CLR_A) property, or the pair (B, T) satisfy (CLR_B) property, then mapping ∧,∈:X→[0,1) such that for all x, y∈X:

∧(Sx)≤∧(x) and Ɇ(Sx)≤Ɇ(x);

⋀(Tx)≤⋀(x)and⋐(Tx)≤⋐(x);

(⋀+⋐) (x) <1;

d(Sx,Sy)≼⋀(x)d(x,y)+⋐(xx)d d(xx,SSxx)d d(yy,TTyy)

1+d(x,y)

Then S and T have unique fixed point.

(iii). Six Maps with A common Fixed point in Complex valued Metric Spaces[22]

In this paper, author attained a common fixed-point theorem for six maps in complete valued metric space which is basically the generalization of [18]

Theorem: Let (X, d) be a complex valued metric space and F, G, I, J, K, L be self-maps of X satisfying the following conditions:

(i). KL(X)⊆F(X) andIJ(X)⊆G(X)

(ii).D(IJx,KLy)≤ ad(Fx,Gy)+b(d(Fx,IJx)+d(gy,KLy))+c(d(Fx,KLy)+d(Gy,IJx)) for all x,y∈X, where a,b,c≥0 and a+2b+2c<1.Assumetatthepair(KL,G)and(IJ,F)areweaklycompatible,pair(K,L),(K,G),(L,G),(I,J),(I,F)and(J,F)arecommutingpairofmaps.ThenK,L,I,J,GandFhaveuniquecommonfixedpointin X.

(iv). Some Common Fixed-Point Result for Rational Type Contraction Mapping in Complex valued Metric Space [23]

In this paper author demonstrate some fixed-point theorem for two pairs which fulfils a rational type condition in complex valued metric space.

Fixed Point Theorem using E.A property

Theorem: Let (X, d) be a complex valued metric space and A, B, S, T:→X for self-mappings satisfying the following conditions:

(i). A(X)⊆T(X), B(X)⊆S(X)

(ii) For all x, y∈X and 0<a<1

d(Ax,By)≤aadd(SSxx,AAxx)dd(SSxx,BByy)+dd(TTyy,BByy)dd(TTyy,AAxx)/1+dd(SSxx,BByy)+dd(TTyy,AAxx) (iii). The pair

(A, S) and (B, T) are weakly compatible.

(iv). One of the pair (A, S) or (B, T) satisfies (E.A)-property,

IftherangeS(X)orT(X)isclosedsubspaceofX,thenthemappingsA,B,SandThaveuniquecommon fixed point inX.

Fixed point theorem using (CLR)-property

IftherangeS(X)orT(X)idsclosedsubspaceofXthenthemappingsA,B,SandThaveuniquecommon fixed point inX.

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Conclusion

I will be very thankful to my professor, Dr. Sangeeta kumari to help me for motivation on research field.
It is concluded that metrics have very important role in the field of fixed-point theory in higher mathematics. Also, we have compared the continuity or uniform continuity in metric space with conversion and point wise convergence.

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