METRIC SPACEAND ITS APPLICATIONS

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ABSTRACT: This paper gives a short review about some basics of metric spaces, concept of continuity of these metric spaces, with comparison of it with uniform continuity and convergence. In addition to this it includes discussion of some past work of researchers on metric space, its generalization and useful results made by them with applications using these spaces.

KEYWORDS: Metricspace, Cauchy sequence, Uniform continuity, Continuous function, Contraction map.

INTRODUCTION

This structure has pulled in an extensive consideration from mathematicians on account of the advancement of the fixed point hypothesis in standard metric space.Generally, concept of metric space is a major apparatus useful in the field of solving some special problems in higher mathematics. As lots of research have been made on this concept of metric spaces. From that point forward, a few works have managed fixed point hypothesis in such spaces. Such, spaces assume a significant part in topology and some of the logical programming based problems and techniques. Basically, in year 1905 metric spaces firstly studied by French famous mathematician named as Maurice Frechet and metric word is related to distance on number line.Here, the metric space which was related to complex condition given or defined by Azam et al which shows that results of fixed point on mapping also follow the condition of inequality which is rationaland defined for that spacesafter which there the generation and some initialization of atheory on cone metric space takes place. After that,other papers have overseen fixed point formation of theory without confirmation with complex esteemed metric space[1],[2].

The hypothesis of particular spaces was started by Nakano then it was reclassified and summed up by MusielakandOrlicz. By characterizing a norm, specific Banach spaces of functions can be thought of as a part of these spaces. Metric fixed hypothesis for these Banach spaces of capacities has been broadly considered and studied.[16]Some typical fixed point results two or three mapping whichfollows the expansive condition of contractioninclude reasonable articulations contains some pointsubordinate control capacities like coefficients involved in the complex valued$b$-metric spaces have been demonstrated by numerous creators.[3]

Banach fixed point hypothesis in a total metric space has been summarized in various spaces. In 2007, R. Bhardwaj in team work with S.S. Rajput and one another member R.N. Yadava has worked for fixed point theorems in complete metric space which help the mathematicians to generalise the results.[3]

LITERATURE REVIEW:

In year 2011, Azam et al.[2]define some of the possibility of complexesteemed metric space and developed adequate conditions and chances for the presence of basic fixed marks of couple of mappings which satisfies important result ofcontractive condition. In 2013, Petko D Proinov developed theory for cone metric space over the solid spaces and describes some new properties on cone metric spaces.

In 2014, N.B. Okelo worked in this field to show by using common fixed point theorem for extensive type of mapping and use these results for information and communication technology[15]. In 2017,Tomonari Suzuki proved basic inequalities and fixed point theorems with two conditions on b-metric space. The chance of the complex-esteemed metric space now can be mishandled in characterize complexesteemed normed space with complexesteemed Hilbertspaces. The hypothesis which was mainly exhibited by Azam et al. and one another Bhatt et al.made reasonable imbalance in complexesteemed metric space and give an important condition known as contractive.

Rao et al. [9] began to focus on fixed point observations and consequences on complex valued*b*-metric space,expansive than complex esteemed metric spaces. Then using some mentioned results of this paper, many creators have shown a couple of results on fixed point for various planning satisfies judicious conditions concerning complex esteemed *b*-metric spaces[4]. Lately, Sintunavaratet al. [6], SitthikulandSaejung [8], and Singhetal. obtained essential results taken from fixedpoint proved to be reliable on the condition of contractionfor control capacities in case of complex esteemed metric space [5].

DEFINITION OF METRIC SPACE

If X as non-empty set. Then metric d, defined on X is function on X×X for all values of x, y and z belongs to X,if it follows these properties:

1. d (x, y) $\geq$ 0 (positivity)
2. d(x,y) = 0 when x=y
3. d(x,y) = d(y,x) (symmetry property)
4. d(x,y) $\leq$ d(x,z)+d(z,y) (triangle inequality)

Hence, when all of these given conditions are satisfied by the function then it is said to be metric space(X,d).

Here, this function may be known as distance function or simply a distance.

EXAMPLES:

- Real numbers for any given function d(x,y)=$|$y-x$|$ is known as absolute difference, but moreover in general language, euclidean n-space defined by Euclidean function, known to be metric spaces as complete.
- Normed vector space becomes metric space when defined as follows d(x,y) =$\|$y-x$\|$, as metrics for vector spaces.
- Complete metric space known as a Banach space.
- Finite metric spaceknown to be finite if contains the finitepoints whereas infinite metric space contains infinite points. Every finitespace cannot becomesatisfied withthe euclidean space.[5]

TYPES OF METRIC SPACE

1. Complete metric spaces:

When every Cauchy sequence converge in given metric space then the metric space becomes complete.In other words, if d$(x_n, y_m) \to 0$when both and individually goes to infinity, and existsany y∈ M which give conditiond$(x_n, y) \to 0$.

As example: d(x,y)=│x-y│, with its absolute valueis not complete space.[4],[5]

2. Compact metric spaces:

When every sequence has particular subsequencewhich converges at a point then metric space said to be compact. Therefore, it also named by sequential compactness but with the case of metric spaces then it becomes similar with the topological theory having countable compactnessthen it is provided through open cover.[7]

3. Locally compact spaces:

A metric space become locallycompact when every given point in that have compact neighbourhood. Theeuclidean spaces also include in such spaces, while the case of infinitedimensional spaces mainly Banach spaces does not called to be locally compact.

Any metric space is defined as proper when every closed ball becomes compact. Here,proper space can be locally compact in generalbut conversepart need not proved to be satisfy.[8],[9]

4. Uniform continuity for Metric spaces:

If there are any two metric spaces $(X, k_1) and (Y, k_2)$, then the function T: X→Y become uniform continuous when for every real number ε > 0 exists some real δ> 0 andk_1(x,y) < δ , there is k_2(T(x),T(y))< ε for x, y∈ X.

Every uniform continuous mapping defined by any function T: X→Y is continuous, but converse of it is only true if X is compact.[10]

5. Lipschitzcontinuous map:

If a real number R> 0, the mapping T: X→Y is lipschitz continuous when satisfied by following condition:

$$d_2(\mathrm{T(x), T(y)}) \leq \mathrm{R} d_1(\mathrm{x, y}) \text{ when x, y} \in \mathrm{X}.$$

Generally, lipschitz continuous mapping always become uniform continuous, conversely it cannot be satisfiiedas truth.

When R< 1, then the function T known as contraction.

Consider the case Y = X with X is complete. Here if T become contraction map the function Tcontain fixed point which is unique.[11],[12]

6. Convergence of Metric spaces:

Let the metric space be (X,d) then the sequence $\{x_n\}$ having points in Xconverges tox$\in$ X when for $\varepsilon > 0$, there exist an integer N satisfies the condition:

$$d(x_n, x) < \varepsilon \text{ when } n>N$$

Here, point x in X knownto be its limit point of that sequence.Every, convergent sequence is also bounded.[13]

APPLICATIONS

Here, the function may be known as distance function or simply a distance. This means that the concept of metric space can be used to solve some distance based problems related to trains, planes, etc. from location A to location B and to find a solution for shortest path used which connects the both locations.Some results of fixedpoint hypothesis become useful in information and communication technology and radiations of satellite transmission. The generalisation results of b-metric spaces become applicable in forensic department, for security purpose of data in computer system and also for higher software computing.[15].Hence, in this way idea of metric space quite applicable in various fields of solving problems related to shortest path between the connecting places.

CONCLUSION

General concepts of metric spaces and most of its generalisation has great extension in various fields of higher mathematics. Basically, it can be understood or named as distance function for solution of path related problems but some results of these can be extended to theory of fixed point also. Its generalisation includes complete spaces, compact and banach spaces. Here, it is also discussed theconcept of continuity of these metric spaces, with comparison of it with uniform continuity and convergence with various forms of these and some basic results provided by metric spaces in research work. Its applications in different fields like to find shortest path, for data security purpose, for signal transmission in communication technology are very useful and popular in higher engineering.

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