STEREOGRAPHIC PROJECTION AND ITS APPLICATIONS

Anjali jangra

Department of Mathematics , Chandigarh University

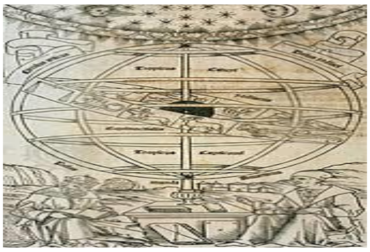
Gharuan, Punjab –India

anjalijangid20000@gmail.com

ABSTRACT :- In this paper we introduce of steriographic projection and its applications. Today's time this is one of the most widely used of stereoprojection. This projection is used in polar areas and also usedin small scale maps. This is very useful tool to solveing the orientation problems in structural geology ,mathematics and also in the studying of crystal structures and helpful to deduce how those structures relate to planes with crystal.

KEYWORDS :- stereographic projection ,3Dplane ,unit sphere , spherical projection,crystalography , photoprojection.

DISCOVERY OF STEREO:- The stereographic projection is given by Hipparchus and Ptolemy. This is originally known as the plano-sphere projection. *PTOLEMY represent sphere in the plane . Plotlemy described that this is the oldest known surviving document and was used to know the movement of stars.*

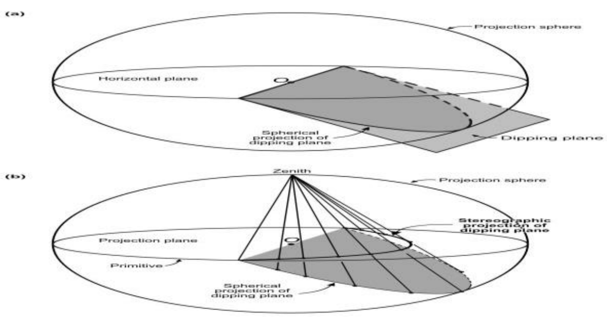

INTRODUCTION :- *It is that type of mapping system that allow us to represent various angle in 3-D space draw on a 1-D paper.* The stereographic projection,in geometry is a particular mapping that projects a sphere onto a plane. The projection is defined on the entire sphere ,except at one point .The projection point. Where it is defined,the mapping is smooth and bijective.it is conformal,meaning that is preserves angles. This is neither isometric nor area preserving ., it preserves neither distance nor the areas of figure. Thus intuitively ,the sterographic projection is a way of picturing the sphere as the plane,with some inevitable compromises. Because the sphere and the plane occur in many areas of mathematics and its application, so does the sterographic projection. It finds use in diverse fields including complexanalysis, cartography, geology,and photography.. Stereographic projection is used for analysis for bedding attitudes ,hinge line , interlimb angles ,orientation of joints , plunge and trend of the intersection of two planes , kinematic analysis for natural and engineering slopes etc.

in practice,the projection is carried out by computer or by hand using a special kind of graph paper called stereonet (or wulffnet). To describe the sterographic projection it is sufficient to set up a one –one mapping .

THE CONCEPT OF PROJECTION :- The earth is approximately spherical the piece of paper or computer screen that displays the map is flat. There is inventably some distoration that occurs when we project a spherical surface onto a flat medium .

There are myrid of map projection that accomplish this task depending on the size of the map area and the purpose for which the map is needed

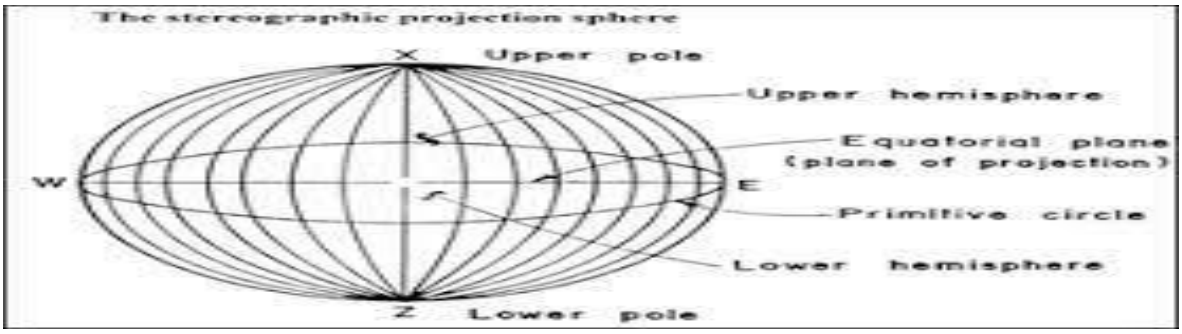
PRINCIPAL OF STEREOGRAPHIC PROJECTION :- In Geology , Stereographic projection are used for primarily to present planar and linear features in 2D diagram and it analyse the mutual relationships between the planer and linear features in different ways. The projection plane is an imaginary horizontal plane passing through the centre of a sphere



PROJECTION OF A LINE :- imagine a linear element passing through the centre of the sphere. The line would intersect the surface of the sphere twice one is upper hemisphere (A) and another in lower hemisphere (B)

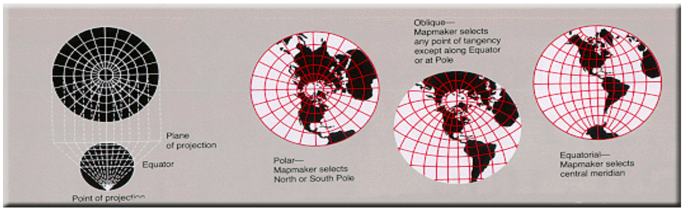
EQUAL AREA NET(SCHMIDT OR LAMBERTNET):- This is required if points are to be contouredinto spatially meaningful concentration. conformal minimizes the area distort ion better analyses the data –accuracy easier for data contouring.

EQUAL ANGLE NET(WALF NET):- An equal angle net produce false concentration from an evently spaced disribution of points although the angular relationships are correct. Equidistant better for kinematic analysis does good job in analysing angular relationships suitable for strain analysis.



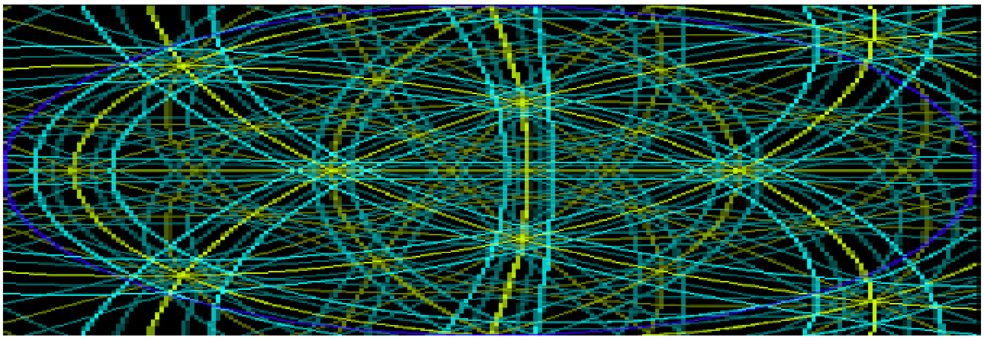
APPLICATIONS OF STEREOGRAPHIC IN DIFFERENT FIELDS:-

- **IN CARTOGRAPHY:-** In his fact that no map from the sphere can accurately represent the both angles and shapes and it also areas in the fundamental problem of cartography. Area preserving map projection are preferred for statistical applications , because they behave good with integration, on the other hand angle preserving map projection are preferred to navigation .stereographic projection falls in second category.



.

IN CRYSTALLOGRAPHY:-A crystal projection is a very quantitative method for representing a three dimensional crystal on a two dimensional planar surface. Crystal projections have some definite rules , so that the projection bears is a known and reproducible relationship with crystal. In the study of crystallography it is o useful to be able to represent the crystal planes and crystal directions on a diagram in two dimensions so that angular relationships and the symmetrical arrangements of crystal faces can be discussed upon a flat piece of paper, and if required for measured. The most useful type of the diagram will be one in which the angular relationships in three dimensions in the crystal, and faithfully reproduced in a plane in some form of projectional geometry.* The conformal projection used in the crystallography is the stereographic projection*

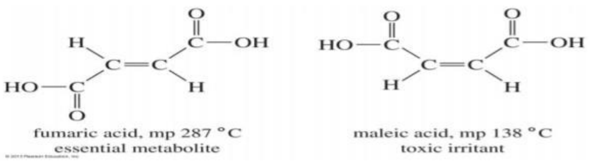


IN GEOLOGY:- It is a very powerful or usefull method for solving the geometric problems in the structural **geology**. To deduce for unlike structure contouring and other map-based techniques, it also preserves the orientation of lines and planes with no ability to preserve position relationships .

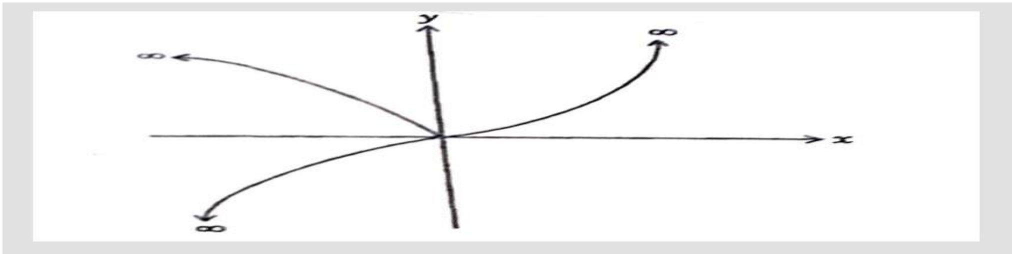
#stereo use in geographic protion as a 1. For landslide hazard/slope failure studies 2. For earthquake studies. 3 . For Structural geologial analysis. 4 . For fracture analyses used in hydrogeology and/or groundwater pollution potenials . 5. For Mining industry (fossil fuels included).

IN PHOTOGRAPHY:- In 3D photography and stereoscopic photography is the art or way of capturing and displaying two slightly different setellment of photographs . To create three dimensional images. The 3D effect is works on the principle is called stereopsis . Each eye of human is in a different location, as a result and also it sees a slightly different image.


IN CHEMISTRY :- In chemistry we use for study of chiral molecules.stereochemistry is also known 3d chemistry .In this we study about stereochemical problems of organic , ingornic,biological ,superamolecular chemistries.In organic chemistry subtle differences in spatial arrangements can give rise to prominent the effects .for example :-



IN MATHEMATICS PROJCTION IN COMPLEX PLANE :-To describe the sterographic projection it is sufficient to set a one –one correspondence between the points on sphere and the points on a plane.



COMPLEX PLANE INFINTY

$z/\infty = 0$;

$z + \infty = \infty (z \neq \infty)$;

$z/0) = \infty \ (z \neq 0)$ $z.\infty = \infty (z \neq 0)$;

Riemann suggested the two methods for construction.

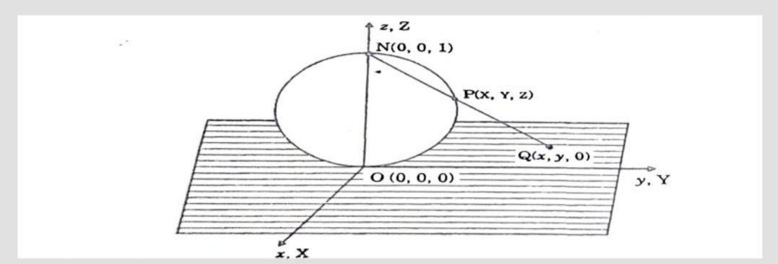
MATHOD I:- Let ρ is the complex plane in this method , we set up a correspondence between the points of the complex plane b and those of a sphere of radius ½ with centreat (0,0,1/2) tangent to this plane.

Let this line be Z-axis of 3-D euclidean space in which a point less coordinates(X , Y, Z).

Consider the sphere S of radius ½ and centre at (0,0, 1/2)

$S = \{(X,Y,Z) \in R^2 : X^2 + Y^2 + (Z - ½)^2 = ¼\}$

Let N (0,0,1) and O (0,0,0) denote the north pole and south pole of the sphere S respectivly. The point z =0 +i.0 coincides with the point O (0,0,0) of the complex plane b and that X and Y axes are the xand y axes respectively .Let Q (x,y,0) utbe any point in the plane b. Corresponding to this point Q(x,y,0) on the complex plane ,there exists a unique point on the sphere S. Through the points N and Q draw a staright line NQ intersecting the sphere S at a point say P(X,Y,Z). Then (x,y,z) is called the steriographic projetion or image of (x,y,0) on the sphere . Now we see that there is a one to one correspondence between the points of and the points of S with one exception namly the north pole (0,0,1) . Let the north pole N of the sphere corresponds to the point at infinity and so we obtain a one to one correspondence between all the points of the sphere S on one hand. This sphere is o called

RIEMAN sphere 

Let Q(x,y,0)(other than infinty) be any point on the complrx plane and its image on sphere S be P(X,Y,Z).

Points (0,0,1) ,(X,Y,Z) and Q(x,y,0) are collinear

$X - 0/x - 0 = Y - 0/y - 0 = Z - 0/z - 0$

⇨ $X/x = Y/y = 1 - Z$ ....(1)
⇨ $X = X/1 - Z$ and $y = Y/1 - Z$ ....(2)
⇨ $x^2 + y^2 = X^2 + Y^2/(1 - Z)^2$
⇨ $= ¼ - (Z - ½)^2 / (1 - Z)^2$

$x^2 + y^2 = Z(1 - Z)/(1 - Z)^2$ ....(3)

As any point P (X,Y,Z) on sphere is different from N (0,0,1)

$Z \neq 1 \Rightarrow 1 - Z \neq 0$

EQ (3) becomes $x^2 + y^2 = Z/(1 - Z)$

Or $(x^2 + y^2)(1 - Z) = Z$

Or $x^2 + y^2 - Z(x^2 + y^2) = Z$

Or $x^2 + y^2 = Z(1 + x^2 + y^2)$

$Z = x^2 + y^2 / 1 + x^2 + y^2$

Here $1 - Z = 1 - x^2 + y^2 / 1 + x^2 + y^2 == 1/1 + x^2 + y^2$ ....(4)

Using (4) in (1) we get ....

$X/x = 1/1 + x^2 + y^2$, $Y/y = 1/1 + x^2 + y^2$

$X = x/1 + x^2 + y^2$, $Y = y/1 + x^2 + y^2$, $Z = x^2 + y^2/1 + x^2 + y^2$

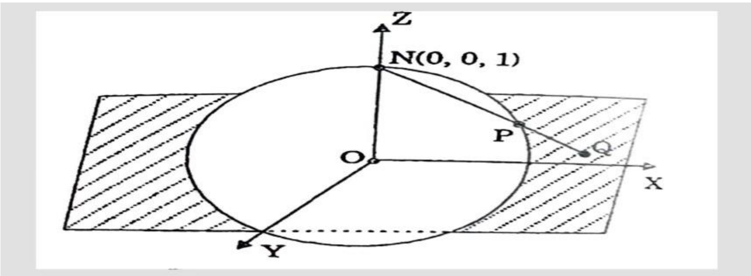
Thus, image of point (x,y,0) on the complex plane is the point (X,Y,Z) on the sphere S, where

$X = x/ 1+ x^2+ y^2$,$Y = y / 1+x^2+y^2$, and $Z = x^2 +y^2/1+x^2+y^2$

METHOD II :- In this method set a correspondence between points of the complex plane. And those of a sphere of radius 1 having centre at (0 , 0 , 0) and the plane passing through (0,0,0). Take any point N on the sphere as usually vertex of projection and its equatorial plane on plane of projection. Any point P of the sphere except the vertex are there corresponds a uniqe point Q of the plane .we construct a line through the origin perpendicular tpo plane . Let this line be Z-axis of a three dimensionl space in which (X,Y,Z) are coordinates of a point . Let S be a sphere with centre (0,0,0) and radius 1.

$S =\{(X,Y,Z) \epsilon \ R^3 : X^2 + Y^2 +Z^2 =1\}$

Through the points N and Q draw a staright line NQ intersecting the sphere S at a point say P(X,Y,Z). Then (x,y,z) is called the steriographic projetion or image of (x,y,0) on the sphere.Now we see that there is a one to one correspondence between the points of and the points of S with one exception namly the north pole (0,0,1) . P(X,Y,Z) on the sphere except north pole N there is a uniqe point on the complex plane . THE north pole of the sphere is the range of the point at infinity in the complex plane.



Points (0,0,1) ,(X,Y,Z) and Q(x,y,0) are collinear

$X – 0/x – 0 = Y – 0/y – 0 = Z – 0/z – 0$

⇨ $X/x = Y/y =1 – Z$ ....(6)

⇨ $X= X /1 –Z$ and $y = Y /1-Z$ ....(7)

⇨ $x^2+ y^2 = X^2 +Y^2/ (1-Z)^2$

$= 1 - Z^2/(1-Z)^2$

$x^2 + y^2 = (1 –Z) (1+Z)/ (1-Z)^2$........(8)

As any point P(X,Y,Z) on the sphere is different from (0,0,1)

$Z \neq 0$ => $1 –Z \neq 0$

FROM (8) we have...

$x^2 + y^2 = 1 + Z / 1\text{-}Z$

$x^2+y^2 - Z(x^2 +y^2) = 1 + Z$

$x^2 + y^2 – 1 = Z(1 + x^2 + y^2)$

$Z = x^2 + y^2 -1 / 1 + x^2 + y^2$

$1- Z = 1 - x^2+ y^2 -1 / 1+ x^2+y^2 = 2 / 1+x^2+y^2$

$X = 2x/ 1+ x^2+ y^2$, and $Y = 2y / 1+x^2+y^2$

Image of point (x,y,0) on the complex planeis the point (X,Y,Z) on the sphere S ,

$X = 2x/ 1+ x^2+ y^2$, $Y = 2y / 1+x^2+y^2$, $Z = x^2 +y^2-1/1+x^2+y^2$

EXAMPLE :- Determine the image or steriographic projection of the given point :-

1 + i

Solution :-x + iy =1 +i

Equating real and imaginary parts , x = 1 and y =1

We know that image of x+iy of complex plane is the point (X,Y,Z) on the sphere,

$X =x/ 1+ x^2+ y^2$,$Y = y / 1+x^2+y^2$, $Z = x^2 +y^2/1+x^2+y^2$

Put x=1 and y =1 then ,

X =1/3 , Y = 1/3 , Z = 2/3

Image of point 1+i of the complex plane is the point (1/3 ,1/3,2/3) on the sphere .

CONCLUSIONS :- Stereoprojection would give us new powerful methimatical tool for research and the combination.The steriographic projection has its origins as a practical tool for the solution to astronomical and navigational problems.By the steriographic had become an indespensible tool in the stdy of optical properties of crystals and in solution of structural problems.

ACKNOWLEDGEMENTS :- I am thankful librarians at The department of mathematics , chandigarh university, university college london, department of geology (kuk), is gratefully acknowledged.

REFERENCES :-

- Bloss,F.D.,1971,Crystallography and crystal Chemistry : Holt , Reinhardt , and Winston,New York,545p.
- British Pharmacopoeia.1993;1:45-46,120-121,132-133.HMSO,London.

- Donn, W. L. and Shimer, J. A. (1958) Graphic Methods in Structural Geology, Appleton Century Crofts, New York.
- Dexter perkins ,2002,Mineralogy,2nd edition Prentice Hall,New Jersey ,483 p
- Goodman, R. E. (1976) Principles of stereographic projection and joint surveys, in Methods of Geological Engineering in Discontinuous Rocks, West, St. Paul, MN.
- Hoek, E. and Bray, J. W. (1977) Graphical presentation of geological data, in Rock Slope Engineering, 2d ed., Institute of Mining and Metallurgy, London.
- Klien, Cornelis and Dutrow, Barbara, (2008). Manual of Mineral Sciences (Manual of Mineralogy), 23rd Edn. John Wiley & Sons, New York. ISBN: 0471721573, 978-0471721574.
- Nesse, William D. (2016). Introduction to Mineralogy, 3rd Edn. Oxford University press. ISBN: 9780190618353.
- Perkins, D. (2015). Mineralogy, 3rd Edn. Pearson Education India. ISBN: 9789332550421, 9332550425.
- Richarard j.Lisle and peter R.Leyshon.
- Wall T and Walle UK: Pharmacokinetic paramacol 1988;3793-6.
- www.oxfordreference.com.,
- http://www.lmcp.jussieu.fr/iucr-top/welcome.html. ,
- http://www.doitpoms.ac.uk/tlplib/stereographic/basic.php ,
- www.tulane.edu.
- http://www.minsocam.org.
- http://dictionary.reference.com/browse/stereo-