Solving Multi-Objective Linear Programming Problems with Fuzzy Interval Based on Decomposition Method

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Abstract

Most of the problems in the real world situations have a multi-objective, in these situations available information in the system are not exact or imprecise. In this paper, solving multi-objective linear programming problems with fuzzy non-negative interval such as objective function coefficients, technical coefficients and fuzzy variables by using an approximation but convenient method called decomposition method has been proposed. In the composition method, ranking functions are not used. With the help of numerical examples, the method is illustrated.

Keywords: multi-objective linear programming, fuzzy interval, fuzzy number, fuzzy linear programming, decomposition method.

1. Introduction

Multi-objective problems occur in many engineering, scientific research projects, and economics, like risk, cost, time minimization or quality, efficiency, and revenue maximization. These are difficult but practical problems which normally happen. Linear programming [2] in many fields has applications of operations research. It is concerned with the optimization of a linear function while satisfying a set of linear equality and/or inequality constraints or restrictions. Multi-objective linear programming has the same assumptions as ordinary linear programming; their objectives and constraint set must be of the =, ≤, or ≥ type, and fractional values for the decision variables are allowed.

In real world situation the available information (data) in the system under consideration are not exact, therefore, fuzzy linear programming (FLP) was introduced and studied by many researchers[14, 15, 5, 4, 8, 10, 9, 12] Fuzzy set theory has become an important tool in the branch of decision making sciences and has been applied to many disciplines such as control theory and management sciences, mathematical modeling and industrial applications.

The concept of fuzzy decision was introduced by [3], later [14] first extended the concept as fuzzy linear programming (FLP) problems on a general level. Fuzzy linear programming problems have an essential role in fuzzy modeling, which can formulate uncertainty in the real world. Afterwards, many researchers have considered various types of FLP problems and proposed several approaches for solving these problems. In effect, the most convenient methods are based on the concept of comparison of fuzzy numbers by use of ranking functions [4, 10, 9]. Usually in such methods, researchers define a crisp model which is equivalent to the FLP problem and then use the optimal solution of the model as the optimal solution of the FLP problem. [11]Use linear ranking functions for solving fuzzy variable linear programming that uses simplex tableau used to solve linear programming problem in crisp environment. [9]Introduced a dual simplex algorithm for solving linear programming problem with fuzzy variables and its dual by using a general linear ranking function and linear programming directly.

All the above researchers considered either the technical coefficients or the variables as fuzzy, and both [1]. Most problems in the real life have multi-objective and sometimes the available information (data) in the system are not exact. Where human estimation is used are inexact, so as the decision that are taken based on this information. Thus, it is desirable to consider the model with multi-objective function and fuzzy coefficients such as objective coefficients, technical coefficients (L.H.S), and variables (R.H.S). The proposed method in this paper, namely decomposition method, used by [13], to solve integer linear programming problem with fuzzy variables, for solving multi-objective linear programming problem with fuzzy objective coefficients, fuzzy technical coefficients and fuzzy variables.

2. Preliminaries

We need the following definitions and theorems to establish the method which can be found at [1].

2.1 Fuzzy Numbers

Definition 1: A fuzzy number α is a convex normalized fuzzy set on the real line R such that:

a. There exists at least one $x_o \in R \; with \; \mu_{\tilde{\alpha}}(x_o) = 1$.
b. $\mu_{\tilde{\alpha}}(x)$ is piecewise continuous

The membership function of any fuzzy number α is as follows:

$$\mu_{\alpha}(x) = \begin{Bmatrix} f_x \;, & x \in a, b \\ 1 \;, & x \in b, c \\ g_x \;, & x \in a, b \\ 0 \;, & otherwise \end{Bmatrix}$$

Where$a \leq b \leq c$, f is increasing and right-continuous function on [a, b], and g is decreasing and left-continuous function on [b, c]. If b = c, then a is a fuzzy number otherwise it is known as fuzzy interval.

Definition 2: A fuzzy number $\bar{a}$ is a Triangular Fuzzy Number (TFN) denoted by a_1, a_2, a_3 where a_1, a_2, a_3 are real numbers and its membership function$\mu_\alpha(x)$ is given below.

$$\mu_\alpha(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, & a_2 \leq x \leq a_3 \\ 0\ , & otherwise \end{cases}$$

Definition 3:A fuzzy number $a = \{x, \mu_\alpha(x), \mathrm{x} \in \mathrm{R}\}$is non-negative and denoted by $a \geq 0$ if and only if $\mu_\alpha(x) = 0$ for all$x < 0$. Then a TFN is non-negative$a = (a_1, a_2, a_3)$ if and only if$a_1 \geq 0$.

Definition 4:A fuzzy matrix $A = [a_{ij}]m\ x\ n$ is called non-negative if $a_{ij} \geq 0$, for all *i*, *j*, where a_{ij}'s are fuzzy numbers.

2.2 Arithmetic on Fuzzy Numbers

Definition 5: Let $a\ and\ b$ be any fuzzy numbers and let * denote any of the four basic arithmetic operations. Then we define a fuzzy set on ℝ, $aA * bB$ by defining its a - cuts, $a_{aA*bB} = a_{aAB} * a_{bB}$ for any$a \in (0,1]$. When * = /, clearly we have to require that$0 \notin a_{bB}\ for\ all\ a \in (0,1]$.

Since a_{aA*bB} is a closed interval for each $a \in (0,1]$ and a, b are fuzzy numbers, $aA * bB$ is also fuzzy numbers, which followed by the following theorem.

Theorem 2.2: Let $* \in \{+, -, \mathrm{x}, /\}$, and let$a, b$denote any continuous fuzzy numbers. Then the fuzzy set$aA * bB$ defined by

$$aA * bB\ z = sup_{z=x*y}\ min[\mu_\alpha(x) * \mu_b(y)]$$

is a continuous fuzzy number.
Proof. Klir and Yuan [7].

Definition 6: Let $a = (a_1, a_2, a_3)$ and $b = (b_1, b_2, b_3)$ be two TFNs.
Then
1) $a + b = (a_1, a_2, a_3) + (b_1, b_2, b_3) = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$
2)$a - b = (a_1, a_2, a_3) + (b_1, b_2, b_3) = (a_1 - b_1, a_2 - b_2, a_3 - b_3)$
3) a)$ka = k(a_1, a_2, a_3) = (ka_1, ka_2, ka_3) for\ k \geq 0$
b)$ka = k(a_1, a_2, a_3) = (ka_3, ka_2, ka_1) for\ k \leq 0$
4)$a.b = (a_1, a_2, a_3)\ .\ (b_1, b_2, b_3) = (a_1.b_1, a_2.b_2, a_3.b_3)a, b$ are non-negative TFN's.
Let $F(R)$ be the set of all real TFNs.

Definition 7: Let $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$ then

- $A = B \Leftrightarrow a_i = b_i\ for\ all\ i = 1\ to\ 3$
- $A \leq B \Leftrightarrow a_i \leq b_i\ for\ all\ i = 1\ to\ 3$

3. MOLP with Fuzzy Interval

Consider the following multi-objective linear programming problem with fuzzy objective coefficients, fuzzy technical coefficients and fuzzy variables

$$(\text{p}_\text{o})\ max\widetilde{z_1} = \widetilde{c_1}\tilde{x}$$
$$max\widetilde{z_2} = \widetilde{c_2}\tilde{x} \qquad (1)$$
$$max\widetilde{z_3} = \widetilde{c_3}\tilde{x}$$

s.t

$$\widetilde{AX} \leq \tilde{b} \qquad (2)$$
$$\tilde{x} \geq 0$$

Where the coefficient matrix $\tilde{A} = (\widetilde{a_{ij}})mxn$ is real fuzzy matrix, and the coefficient vector $\tilde{C} = (\widetilde{c_j})nx1$ is a non-negative fuzzy vector, and $\tilde{X} = (\widetilde{x_j})nx1$ and $\tilde{b} = (\widetilde{b_i})mx1$ are a non-negative real fuzzy vectors such that $\widetilde{X_J}, \widetilde{b_i} \in f(R)\ for\ all\ 1 \leq j \leq n\ and\ 1 \leq i \leq m$

Let objective1 $Z_A = \widetilde{c_1}\tilde{x}$, objctive 2 $Z_B = \widetilde{c_2}\tilde{x}$, objctive 3 $Z_C = \widetilde{c_3}\tilde{x}$, Z_T is the new objective function after transformed multi-objective functions into a single objective by used weighted method.Supposed that the weighting coefficients $\mathbf{w_i}$ are real numbers such that $\mathbf{w_i} \geq 0$. Then the single objective will become as follows

$$(\text{P})\ Z_T = [w_1 Z_A + w_2 Z_B + w_3 Z_C]X$$
$$AX \leq b \quad (3)$$
$$X \geq 0$$
$$\text{where}\ w_1 + w_2 + w_3 = 1 \quad (4)$$
$$w_1, w_2, w_3 \geq 0 \qquad (5)$$

(Theorem 2.2.3. and the proof of weighted method in Kaisa Miettinen [6])

Definition 8: A fuzzy vector $\tilde{x}$ is said to be a feasible solution of the problem (P) if $\tilde{x}$ satisfies (3).

Definition 9: A feasible solution $\tilde{x}$ of the problem (P) is said to be an optimal solution of the problem (P) if there exists no feasible $\tilde{u} = (\widetilde{u_j})nx1$ of (P) such that $c\tilde{u} > c\tilde{x}$.

After used the theorem 1 in[13] and the arithmetic operations of fuzzy numbers, we can obtain the following result.

Theorem3.1: A fuzzy vector $\tilde{x} = (x_1^o, x_2^o, x_3^o)$ is an optimal solution of the problem (P) if and only if $x_1^o, x_2^o, and\ x_3^o$ are optimal solutions of the following crisp linear programming problems (P2), (P1) and (P3) respectively where

$$(\text{P2})\ Z_{T2} = [w_1 Z_{A2} + w_2 Z_{B2} + w_3 Z_{C2}]X_2$$
$$A^2 X_2 \leq b_2$$
$$X_2 \geq 0$$
$$\text{where}\ w_1 + w_2 + w_3 = 1$$
$$w_1, w_2, w_3 \geq 0$$

$$(\text{P1})\ Z_{T1} = [w_1 Z_{A1} + w_2 Z_{B1} + w_3 Z_{C1}]X_1$$
$$A^1 X_1 \leq b_1$$
$$X_1 \leq X_2$$
$$X_1 \geq 0$$
$$\text{where} \quad w_1 + w_2 + w_3 = 1$$
$$w_1, w_2, w_3 \geq 0$$

$$(\text{P3})\ Z_{T3} = [w_1 Z_{A3} + w_2 Z_{B3} + w_3 Z_{C3}]X_3$$
$$A^3 X_3 \leq b_3$$

$$X_3 \geq X_2$$
$$X_3 \geq 0$$
$$\text{where} w_1 + w_2 + w_3 = 1$$
$$w_1, w_2, w_3 \geq 0$$

Where $A^K = a_{ij}^K$, $k = 1,2,3$

Proof: the problem (p_o) can be written in the following form

$$\text{(AP)} \max(Z_{T1}, Z_{T2}, Z_{T3}) = [(w_1 C_1^1 + w_2 C_2^1 + w_3 C_3^1)X_1, (w_1 C_1^2 + w_2 C_2^2 + w_3 C_3^2)X_2, (w_1 C_1^3 + w_2 C_2^3 + w_3 C_3^3)X_3]$$

S.T

$$(A^1 X^1, A^2 X^2, A^3 X^3) \leq (b^1, b^2, b^3)$$
$$x_1, x_2, x_3 \geq 0$$
$$A^1, A^2, A^3 \geq 0$$
$$\text{Where } w_1 + w_2 + w_3 = 1$$
$$w_1, w_2, w_3 \geq 0$$

Let $x^o = (x_1^o, x_2^o, x_3^o)$be an optimal solution of the problem (AP) and $x = (x_1, x_2, x_3)$be a feasible solution of the problem (P). This implies that

$$(w_1 C_1^1 + w_2 C_2^1 + w_3 C_3^1)X_1 \leq (w_1 C_1^1 + w_2 C_2^1 + w_3 C_3^1)X_1^O$$
$$(w_1 C_1^2 + w_2 C_2^2 + w_3 C_3^2)X_2 \leq (w_1 C_1^2 + w_2 C_2^2 + w_3 C_3^2)X_2^O$$
$$(w_1 C_1^3 + w_2 C_2^3 + w_3 C_3^3)X_3 \leq (w_1 C_1^3 + w_2 C_2^3 + w_3 C_3^3)X_3^O$$
$$(A^1 X^1 \leq b^1, A^2 X^2 \leq b^2, A^3 X^3 \leq b^3)\,; x_1^o, x_2^o, x_3^o \geq 0 \qquad (6)$$

$$\text{Where } w_1 + w_2 + w_3 = 1$$
$$w_1, w_2, w_3 \geq 0$$

$$\max Z_{T1} = (w_1 C_1^1 + w_2 C_2^1 + w_3 C_3^1)X_1^O; \max Z_{T2} = (w_1 C_1^2 + w_2 C_2^2 + w_3 C_3^2)X_2^O; \max Z_{T3} = (w_1 C_1^3 + w_2 C_2^3 + w_3 C_3^3)X_3^O \qquad (7)$$

Now, from (6) and (7) we can conclude thatx_1^o, x_2^o *and* $x_3^o$are optimal solutions of the crisp LP problems (AP2), (AP1) and (AP3).

Suppose that x_1^o, x_2^o *and* x_3^o are optimal solutions of the crisp LP problems (AP2), (AP1) and (AP3) with optimal values z_1, z_2 *and* z_3 respectively. This implies that $x^o = (x_1^o, x_2^o, x_3^o)$is an optimal solution of the problem (AP) with optimal value $z^o = (z_1^o, z_2^o, z_3^o)$and since (AP) is approximately equivalent to (P), therefore $x^o = (x_1^o, x_2^o, x_3^o)$ is an approximate optimal solution of the problem (P) and hence the result.

4. Algorithm and Methodology

Consider the MOLP problem (p_o) with fuzzy objective coefficients, fuzzy technical coefficients and fuzzy variables

Step 1:choose $w_i$where $\sum_{i=1}^n w_i = 1$, $w_i \geq 0$ depends on the number of objective functions, then construct crisp LP problem

$$\text{(P2) } Z_{T2} = [w_1 C_1^2 + w_2 C_2^2 + w_3 C_3^2]x^2$$
$$A^2 x^2 \leq b^2$$
$$x^2 \geq 0$$

and let x^{2^o} be optimal solution of the problem (P2)

Step 2: construct crisp LP problem

$$(\mathrm{P1})\ Z_{T1} = [w_1C_1^1 + w_2C_2^1 + w_3C_3^1]x^1$$
$$A^1x^1 \leq b^1$$
$$x^1 \leq x^2$$
$$x^1 \geq 0$$

and let x^{1^o} be optimal solution of the problem (P1)

Step 3: construct crisp LP problem

$$(\mathrm{P3})\ Z_{T3} = [w_1C_1^3 + w_2C_2^3 + w_3C_3^3]x^3$$
$$A^3x^3 \leq b^3$$
$$x^3 \geq x^2$$
$$x^3 \geq 0$$

and let x^{3^o} be optimal solution of the problem (P3)

Then the optimal solution of the original ($\mathrm{p_o}$) problem is $x^o = \left(x^{1^o}, x^{2^o}, x^{3^o}\right)$.

5. Numerical Examples

To illustrate this method two numerical examples are presented

5.1 Example 1

$$(p)\max Z_A = [5,8,12]x_1 + [6,9,15]x_2$$
$$\max Z_B = [8,13,18]x_1 + [12,19,25]x_2$$
$$\max Z_C = [12,16,22]x_1 + [18,22,28]x_2$$

s.t

$$[10,13,15]x_1 + [13,16,20]x_2 \leq [200,325,480]$$
$$[8,10,13]x_1 + [28,31,37]x_2 \leq [350,520,735]$$
$$x_1, x_2 \geq 0$$

Let $\widetilde{Z_T} = (Z_{T1}, Z_{T2}, Z_{T3}), \widetilde{x_1} = (y_1, x_1, t_1), \widetilde{x_2} = (y_2, x_2, t_2)$

Now the problem (p2) is given blow with ($w_1 = \frac{1}{3}$, $w_2 = \frac{1}{3}$, $w_3 = \frac{1}{3}$)

$$(p2)\max Z_{A2} = (8x_1 + 9x_2)\frac{1}{3}$$
$$\max Z_{B2} = (13x_1 + 19x_2)\frac{1}{3}$$
$$\max Z_{C2} = (16x_1 + 22x_2)\frac{1}{3}$$
$$Z_{T2} = (8x_1 + 9x_2)\frac{1}{3} + (13x_1 + 19x_2)\frac{1}{3} + (16x_1 + 22x_2)\frac{1}{3}$$
$$Z_{T2} = 12.33x_1 + 16.67x_2$$

s.t

$$13x_1 + 16x_2 \leq 325$$
$$10x_1 + 31x_2 \leq 520$$
$$x_1, x_2 \geq 0$$

Now, using an algorithm for LP problem, the solution of the problem (p2) is

$$\boldsymbol{x_1 = 7.22}\,, \qquad \boldsymbol{x_2 = 14.44}\,, \qquad \boldsymbol{Z_{T2} = 329.84}$$

Now, the problem (p1) is given below

$$(p1)\max Z_{A1} = (5y_1 + 6y_2)\frac{1}{3}$$
$$\max Z_{B1} = (8y_1 + 12y_2)\frac{1}{3}$$
$$\max Z_{C1} = (12y_1 + 18y_2)\frac{1}{3}$$
$$Z_{T1} = (5y_1 + 6y_2)\frac{1}{3} + (8y_1 + 12y_2)\frac{1}{3} + (12y_1 + 18y_2)\frac{1}{3}$$
$$Z_{T1} = 8.33y_1 + 12y_2$$

s.t

$$10y_1 + 13y_2 \le 325$$
$$8y_1 + 28y_2 \le 520$$
$$y_1 \le 7.22$$
$$y_2 \le 14.44$$
$$y_1, y_2 \ge 0$$

Now, using an algorithm for LP problem, the solution of the problem (p1) is

$$\boldsymbol{y_1 = 5.97}\,, \qquad \boldsymbol{y_2 = 10.8}\,, \qquad \boldsymbol{Z_{T1} = 179.24}$$

Now, the problem (p3) is given below

$$(p3)\max Z_{A3} = (12t_1 + 15t_2)\frac{1}{3}$$
$$\max Z_{B3} = (18t_1 + 25t_2)\frac{1}{3}$$
$$\max Z_{C3} = (22t_1 + 28t_2)\frac{1}{3}$$
$$Z_{T3} = (12t_1 + 15t_2)\frac{1}{3} + (18t_1 + 25t_2)\frac{1}{3} + (22t_1 + 28t_2)\frac{1}{3}$$
$$Z_{T3} = 17.33t_1 + 22.67t_2$$

s.t

$$15t_1 + 20t_2 \le 480$$
$$13t_1 + 37t_2 \le 735$$
$$t_1 \ge 7.22$$
$$t_2 \ge 14.44$$
$$t_1, t_2 \ge 0$$

Now, using an algorithm for LP problem, the solution of the problem (p3) is

$$\boldsymbol{t_1 = 12.75}\,, \qquad \boldsymbol{t_2 = 14.44}\,, \qquad \boldsymbol{Z_{T3} = 548.25}$$

Therefore, the solution for the given MOLP problem with fuzzy objective coefficients, fuzzy technical coefficients and fuzzy variables is

$$\widetilde{x_1} = (y_1, x_1, t_1) = (5.97, 7.22, 12.75)$$
$$\widetilde{x_2} = (y_2, x_2, t_2) = (10.8, 14.44, 14.44)$$
$$\widetilde{Z_T} = (Z_{T1}, Z_{T2}, Z_{T3}) = (179.24, 329.84, 548.25)$$

5.2 Example 2

$$(p)\max Z_A = [3,6,12]x_1 + [4,8,13]x_2$$
$$\max Z_B = [5,8,14]x_1 + [3,7,12]x_2$$

$$\min Z_C = [2,6,9]x_1 + [4,7,9]x_2$$

s.t

$$[5,9,15]x_1 + [4,7,10]x_2 \le [40,117,270]$$
$$[4,7,13]x_1 + [14,17,21]x_2 \le [100,207,420]$$
$$x_1, x_2 \ge 0$$

Let $\widetilde{Z_T} = (Z_{T1}, Z_{T2}, Z_{T3}), \widetilde{x_1} = (y_1, x_1, t_1), \widetilde{x_2} = (y_2, x_2, t_2)$

Now the problem (p2) is given blow with $(w_1 = \frac{1}{3}, w_2 = \frac{1}{3}, w_3 = \frac{1}{3})$

$$(p2) \max Z_{A2} = (6x_1 + 8x_2)\frac{1}{3}$$
$$\max Z_{B2} = (8x_1 + 7x_2)\frac{1}{3}$$
$$\min Z_{C2} = (6x_1 + 7x_2)\frac{1}{3}$$
(Must transform Z_{C2} to max) $\max Z_{C2} = (-6x_1 - 7x_2)\frac{1}{3}$

$$(all\ \max) Z_{T2} = (6x_1 + 8x_2)\frac{1}{3} + (8x_1 + 7x_2)\frac{1}{3} + (-6x_1 - 7x_2)\frac{1}{3}$$
$$Z_{T2} = 2.67x_1 + 2.67x_2$$

s.t

$$9x_1 + 7x_2 \le 117$$
$$7x_1 + 17x_2 \le 207$$
$$x_1, x_2 \ge 0$$

Now, using an algorithm for LP problem, the solution of the problem (p2) is

$$\boldsymbol{x_1 = 5.19}, \quad \boldsymbol{x_2 = 10.04}, \quad \boldsymbol{Z_{T2} = 40.67}$$

Now, the problem (p1) is given below

$$(p1) \max Z_{A1} = (3y_1 + 4y_2)\frac{1}{3}$$
$$\max Z_{B1} = (5y_1 + 3y_2)\frac{1}{3}$$
$$\min Z_{C1} = (2y_1 + 4y_2)\frac{1}{3}$$
$$\max Z_{C1} = (-2y_1 - 4y_2)\frac{1}{3}$$

$$(all\ \max) Z_{T1} = (3y_1 + 4y_2)\frac{1}{3} + (5y_1 + 3y_2)\frac{1}{3} + (-2y_1 - 4y_2)\frac{1}{3}$$
$$Z_{T1} = 2y_1 + y_2$$

s.t

$$5y_1 + 4y_2 \le 40$$
$$4y_1 + 14y_2 \le 100$$
$$y_1 \le 5.19$$
$$y_2 \le 10.04$$
$$y_1, y_2 \ge 0$$

Now, using an algorithm for LP problem, the solution of the problem (p1) is

$$\boldsymbol{y_1 = 5.19}, \quad \boldsymbol{y_2 = 3.51}, \quad \boldsymbol{Z_{T1} = 13.89}$$

Now, the problem (p3) is given below

$$(p3)\ \max Z_{A3} = (12t_1 + 13t_2)\frac{1}{3}$$
$$\max Z_{B3} = (14t_1 + 12t_2)\frac{1}{3}$$
$$\min Z_{C3} = (9t_1 + 9t_2)\frac{1}{3}$$
$$\max Z_{C3} = (-9t_1 - 9t_2)\frac{1}{3}$$

$$Z_{T3} = (12t_1 + 13t_2)\frac{1}{3} + (14t_1 + 12t_2)\frac{1}{3} + (-9t_1 - 9t_2)\frac{1}{3}$$
$$Z_{T3} = 5.67t_1 + 5.33t_2$$

s.t

$$15t_1 + 10t_2 \leq 270$$
$$13t_1 + 21t_2 \leq 420$$
$$t_1 \geq 5.19$$
$$t_2 \geq 10.04$$
$$t_1, t_2 \geq 0$$

Now, using an algorithm for LP problem, the solution of the problem (p3) is

$$\boldsymbol{t_1 = 7.95}, \quad \boldsymbol{t_2 = 15.08}, \quad \boldsymbol{Z_{T3} = 125.44}$$

Therefore, the solution for the given MOLP problem with fuzzy objective coefficients, fuzzy technical coefficients and fuzzy variables is

$$\widetilde{x_1} = (y_1, x_1, t_1) = (5.19, 5.19, 7.95)$$
$$\widetilde{x_2} = (y_2, x_2, t_2) = (3.51, 10.04, 15.08)$$
$$\widetilde{Z_T} = (Z_{T1}, Z_{T2}, Z_{T3}) = (13.89, 40.67, 125.44)$$

6. Conclusion

Multi-objective linear programming problems with fuzzy interval are considered in this paper. A simple and efficient method can solve such problems. This method can solve problems under consideration by transforming a linear multi - objective to linear single objective and provide an approximation to the optimal solution without using ranking functions and applying classical single linear programming technique. If the multi-objective problem with fuzzy numbers is transforming to the single objective and crisp, the method produced three crisp linear programming problems with the same optimum solutions.

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