Solving Multi-Objective Transportation Problem Using Summation Penalty Method

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Abstract: *This paper introduces a new Summation Method (SM) for solving Multi-Objective Transportation Problem (MOTP). In this method, the multi-objective transportation problem converted into a single objective transportation problem, where each cost in the single objective transportation problem table refers to the summation of the costs in the objectives in the same location of the cell. The problem solved by TORA using North West Corner (NWC) method to solve the problem. The method is very easy to apply, and provides a non-dominated solution at each iteration which identified by goal programing approach and interactive approach and other methods.*

Keywords: Multi-objective, North West Corner method, Transportation problem, Summation Method.

1. Introduction

Transportation problem is a special class of linear programming problem which deals with the distribution of the commodity from sources to destination. To obtain the initial basic feasible solution, there are several methods as NWC method, Least Cost (LC) method, and the Vogel's approximation method (VAM). The best method of them is the Vogel's approximation method (VAM), which yields the best initial basic feasible solution. The transportation problem in this case is aim to single objective. But often in real life problem, there are multiple objectives needed to achieve while making the transportation operation.

There are several approaches proposed for solving MOTP. Bit (1992) presented an application of fuzzy linear programming to the linear multi-objective transportation problem. Youness (1995) proposed a new approach for finding all efficient solutions for the vector optimization problem without using any parameters. Abd El-Wahed (2001) presented a fuzzy programming approach to determine the optimal compromise solution to multi-objective transportation problem. Abd El Wahed and Sang (2006) presented an interactive fuzzy goal programming approach to determine the preferred compromise solution for the multi-objective transportation problem. Pandian (2011) proposed a new method namely, dripping method to obtain a set of efficient solutions to a bi-objective transportation problem. Anuradha (2012) proposed a new approach, named sum of objectives (SO) method to obtain a fair solution to multi-objective programming problems. Abdul Quddoos et al. (2013) proposed a new method, namely the MMK-method for finding non-degenerate compromise optimal solution for Bi-objective transportation problem (BTP). Anukokila et al. (2017) presented a goal programming approach for solving multi-objective transportation problem. Mohammad et al. (2017) proposed a weighted approach based on goal programming to obtain the compromise solutions to the multi-objective transportation problem. Afwat et al. (2018) proposed a new method called (product approach)to solve multi-objective transportation problem, where they used the fuzzy programming to convert the objectives which have different units to membership value then aggregate them by product. LakhveerKaur and Sandeep (2018) proposed a simple approach to obtain unique efficient solution directly for linear multi-objective transportation problem (MOTP), which is preferred by decision maker. Patel (2018) proposed a new method named row maxima method to solve the multi-objective transportation problem. Rizk et al. (2018) developed a new compromise algorithm for multi-objective transportation problem, which is inspired by Zimmermann's fuzzy programming. Anuradha et al. (2019) proposed a row maxima method to solve bi-objective solid transportation problem (BOSTP) using fuzzy linear membership function.

This paper will be organized as follows: in section 2 presents a short review including the mathematical model to MOTP. Section 3 considers the proposed summation method. Section 4 shows numerical example for solving MOTP using the proposed summation method. Finally, conclusion is given in Section 5.

2. Multi Objective Transportation Problem

In multi objective transportation problem, the transportation problem has more than one conflict objectives, as minimizing the time and the cost or other objectives. The mathematical model of multi-objective transportation problem is as follows [6]:

$$min F^r = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^r x_{ij} \quad (1)$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i, \qquad \forall \quad i = 1,2,\dots\dots m. \quad (2)$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \qquad \forall \quad j = 1,2,\dots\dots n. \quad (3)$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \quad and x_{ij} \geq 0, \forall i, \quad (4)$$

where: a_i represent the units available at source (i). b_i represent the units demanded at destination (j). x_{ij} represent the unknown quantity shipped from source (i) to destination (j). c_{ij} represent the cost of shipping units from source (i) to destination (j). F^r represent the a vector of r objective functions, and $F^r = F^1(x), F^2(x), \dots . F^r(x)$.

3. Steps Of Proposed Method

This paper presents a new method to solve multi-objective transportation problem (MOTP), through sum the objectives and convert them to a single objective, then solve the problem by any package and the set of non-dominated solution provided by each iteration of solution. Steps of proposed method are as following:

1. Convert the MOTP into SOTP where the penalty in each cell equals the summation of penalties in the same location of the cell in other objectives.

2. Identify the initial basic feasible solution by using NWC method (because we need big number of steps to own a chance to allocate as much as of non-dominated solutions), and apply the solution on all objectives.

3. Record in the compromise solution's set both of the objective value for each objective and the value of all objectives together.

4. Check the optimality of the initial feasible solution for SOTP by using modified distribution method (MODI method), and after each iteration record in the compromise solution's set both of the objective value for each objective and the value of all objectives together, until reach the optimal.

5. Stop and present the set of compromise solutions to the decision-maker.

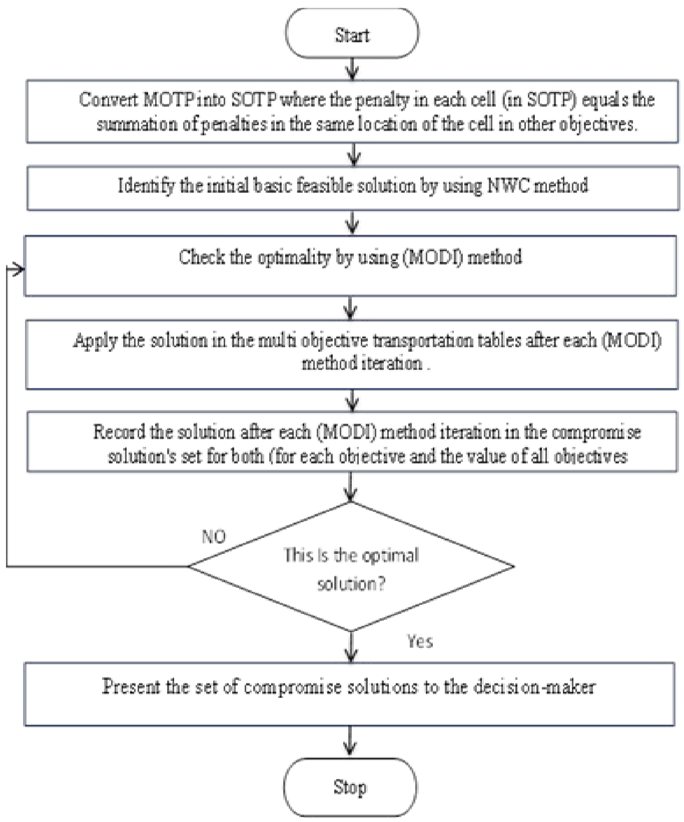

Figure 1: Steps of Summation Method for Solving Multi Objective Transportation Problem.

4. Numerical Example

The following example which solved by several methods will solve by the proposed method (Summation method). The following three objectives transportation problem is:

$$F1 \begin{bmatrix} 9 & 12 & 9 & 6 & 9 \\ 7 & 3 & 7 & 7 & 5 \\ 6 & 5 & 9 & 11 & 3 \\ 6 & 8 & 11 & 2 & 2 \end{bmatrix}, F2 = \begin{bmatrix} 2 & 9 & 8 & 1 & 4 \\ 1 & 9 & 9 & 5 & 2 \\ 8 & 1 & 8 & 4 & 5 \\ 2 & 8 & 6 & 9 & 8 \end{bmatrix}, \quad F3 = \begin{bmatrix} 2 & 4 & 6 & 3 & 6 \\ 4 & 8 & 4 & 9 & 2 \\ 5 & 3 & 5 & 3 & 6 \\ 6 & 9 & 6 & 3 & 1 \end{bmatrix}$$

With supplies a_1 =5, a_2 =4,$a_3 = 2, a_4$ =9, and demands b_1 =4,b_2 =4, b_3 =6, b_4 =2, b_5 =4.

Table 1. Represent the convert of multi-objective to single objective

	D1	D2	D3	D4	D5	supply
S1	13	25	23	10	19	5
S2	12	20	20	21	9	4
S3	19	9	22	18	14	2
S4	14	25	23	14	11	9
Demand	4	4	6	2	4	

Table 2. Represent the initial basic feasible solution by using NWC

	D1	D2	D3	D4	D5	supply	
S1	13	25	23	10	19	5	
	4	1					
S2	12	20	20	21	9	4	
		3	1				
S3	19	9	22	18	14	2	
			2				
S4	14	25	23	14	11	9	
			3	2	4		
Demand	4	4	6	2	4		342

Table 3. Represent iteration 2

	D1	D2	D3	D4	D5	supply	
S1	13	25	23	10	19	5	
	4	1					
S2	12	20	20	21	9	4	
		1	3				
S3	19	9	22	18	14	2	
		2					
S4	14	25	23	14	11	9	
			3	2	4		
Demand	4	4	6	2	4		316

Table 4. Represent iteration 3

	D1	D2	D3	D4	D5	supply
S1	13	25	23	10	19	5
	4			1		
S2	12	20	20	21	9	4
		2	2			
S3	19	9	22	18	14	

		2				2	
S4	14	25	23	14	11	9	
			4	1	4		
Demand	4	4	6	2	4		310

Table 5.Represent iteration 4

	D1	D2	D3	D4	D5	supply	
S1	13	25	23	10	19	5	
	3			2			
S2	12	20	20	21	9	4	
		2	2				
S3	19	9	22	18	14	2	
		2					
S4	14	25	23	14	11	9	
	1		4		4		
Demand	4	4	6	2	4		307

Table 6. Represent the solution at all objectives

	F1					
	D1	D2	D3	D4	D5	
S1	9	12	9	6	9	5
	3			2		
S2	7	3	7	7	5	4
		2	2			
S3	6	5	9	11	3	2
		2				
S4	6	8	11	2	2	9
	1		4		4	
	4	4	6	2	4	
					127	

	F2					
	D1	D2	D3	D4	D5	
S1	2	9	8	1	4	5
	3			2		
S2	1	9	9	5	2	4
		2	2			
S3	8	1	8	4	5	2
		2				
S4	2	8	6	9	8	9
	1		4		4	
	4	4	6	2	4	
					104	

	F3						
	D1	D2	D3	D4	D5		
S1	2	4	6	3	6	5	
	3			2			
S2	4	8	4	9	2	4	
		2	2				
S3	5	3	5	3	6	2	
		2					
S4	6	9	6	3	1	9	
	1		4		4		
	4	4	6	2	4		
					76		307

Table7.The results of the famous methods which solve the example

	Ideal Solution	Net Deviation Approach Mouli et al. (2005)	Interactive Approach Ringuest and Rinks (1987)	Fuzzy Approach Waiel(2001)
F1	102	127	127	122
F2	73	104	104	106
F3	64	76	76	80
$\sum F(X)$		307	307	308

In the previous tables, the proposed method provides a set of solutions in each iteration and is compared with others and it is equal to two of them and different to one of them.

5. Conclusion

There are many methods are presented to solve Multi-Objective Transportation Problem (MOTP). In this paper, a new method proposed to solve MOTP. This method provides a set of solutions in each iteration as a result of initial solution by using MODI method. The method is very simple and provides a set of non-dominated solutions. The proposed method is compared with others and it is equal to two of them and different to one of them.

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