Some Recent Developments in theory of Metric Spaces

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ABSTRACT: The topology on a set X is formed by a non-negative real valued scalar function called metric, which may be understood as measuring some quantity. Because some of the set's attributes are similar, there's a distance between any two elements, or points. Quite evocative of the common concept of distance that we come across in our daily lives. Because its topology is entirely defined by a scalar distance function, this sort of topological space has a distinct advantage over all others. We may reasonably assume that we are familiar with the qualities of such a function and are capable of dealing with it successfully. Instead, a generic topology is frequently dictated by a set of perhaps abstract rules. Frecklet initially proposed the concept of a metric space in 1906, but it was hausdroff who coined the phrase metric space a few years later.

INTRODUCTION

A metric space is a set that has a metric on it in mathematics. A metric is a function that measures something provide a distance concept between any two components of the set, which are commonly referred to as points. 'The'metric meets a few criteria.

Informally, the distance between A and B is zero if and only if A and B are the same point, the distance between two separate points is positive, and the distance between two separate points is negative.

• The distance between points A and B is the same as the distance between points B and A, and

• The distance between points A and B is less than or equal to the distance between points A and B through any third point C.

Topological qualities such as open and closed sets are induced by a metric on a space, leading to the study of more abstract topological space. The three-dimensional Euclidean space is the most well-known metric space. In reality, a metric is an extension of the Euclidean metric, which is defined by four well-known features.The distance between two places expressed as the length of a section of a straight line connecting them. Other metric spaces may be found in elliptic and hyperbolic geometry, where a metric is a distance on a sphere measured by an angle, and special relativity uses the hyperboloid model of hyperbolic geometry as a metric space of velocity.

HISTORY

In his paper sobre sqelques pointsdu calcul Fonctionnel, Maurice Frechet proposed metric spaces in 1906. Felix Hausdorff, however, is responsible for the name.

Types of metric space

- **Complete spaces**
- **Bounded and totally bounded spaces**
- **Compact spaces**
- **Locally compact and proper spaces**
- **Connectedness**
- **Separable spaces**
- **Pointed metric spaces**

Generalizations of metric spaces:

In a natural way, every metric space is a uniform space, and every uniform space is a topological space. As a result, uniform and topological spaces can be considered extensions of metric spaces.

• If you take the first definition of a metric space and waive the second criteria, you get the notions of a pseudo metric space or a dislocated metric space. We get a quasimetric space or a semi metric space if we delete the third or fourth.

• If a distance function accepts values from the extended real number line but otherwise meets all Four requirements, it is referred to be an extended metric, and the accompanying space is referred to as a metric Space.

• Approach spaces are metric spaces that are based on point-to-set distances rather than point-to-point lengths.

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- A continuity space is a generalization of metric spaces and posts ,that can be used to unify the notions of metric spaces and domains.

• A partial metric space is the most extension of the concept of a metric space, with each point's distance from itself no longer having to be zero.

Metric space's topological qualities

Metric space is non-contiguous. Normal spaces are Hausdorff spaces (indeed they are perfectly normal). Any metric space permits partitions of unity, and every Continuous real-valued function defined on a closed subset of a metric space may be extended to a Continuous map on the entire space. Every real valued lipschitz-continuous map on the whole space is also true.

Metric spaces are countable at first because rational-radius balls may be used as a neighbourhood base.

Quantum mechanics applications of metric space

The qualities of two different types of mathematical spaces are combined in Hilbert space: vector space and metric space. While the vector space features are commonly employed, the metric space features are far less so. We show that in quantum mechanics, conservation principles naturally lead to metric spaces for a collection of linked physical quanties. The geometry of all such metric spaces is called a "onion shell.". The associated metric divided focus space into concentric circles, allowing maximum and minimum distances between states to be determined and geometrically understood. Unlike the standard Hilbert's-space analysis, our results are unique. Apply the reduced space of any ground state and partial densities, which are metric but not Hilbert space, as well.

The Hohenberg-Kohn mapping between densities and ground states, which is exceedingly complicated and nonlocal in coordinate descriptions, is shown to be simple in metric space, where it becomes a monotonic and almost linear mapping of vicinities onto vicinites for the system under consideration

Similarly, we investigate the mapping between wave function and (current and partical)densities at the Core of current density functional theory by studying metric spaces associated with multiple body systems submerged in magnetic field. We discover a “band structure” in the linked metric spaces areas of permissible and prohibited distancs, which arises directly from the conservation of the z component of the angular momentum. Finally, new findings contain an acceptable gauge for external potential, allowing us to directly investigate the Hohenberg-"third Kohntheorem's led."

NOTIONS OF METRIC SPACE EQUIVALENCE

The topology on a set X is formed by a non-negative real valued scalar function called metric, which may be understood as measuring some quantity.Because some of the set's attributes are similar, there's a distance between any two elements, or points.Quite evocative of the common concept of distance that we come across in our daily lives. Because its topology is entirely defined by a scalar distance function, this sort of topological space has a distinct advantage over all others. We may reasonably assume that we are familiar with the qualities of such a function and are capable of dealing with it successfully. Instead, a generic topology is frequently dictated by a set of perhaps abstract rules. Frecklet initially proposed the concept of a metric space in 1906, but it was hausdroff who coined the phrase metric space a few years later.

Let (X,) be a partly ordered set, and suppose that there exists a metric d in X such that (X, d) is a full metric space, as stated by **Harjani and Sadarangani (2010).** Let T: X X be a nondecreasing mapping such that (d(Tx, Ty)) d(x, y) d(x, y) d(x, y) d(x, y) d(x, y) d(x, y) d(x, y) d(x, y) d(x, y) d(x, y) d(x, Assume that either (I) T is continuous or (I) T is discontinuous (II) If xn X is a nondecreasing sequence with xn x X, then for every n, xn x. If x0 X exists and x0 Tx0 exists, T has a fixed point.

Eslamian and Abkar suggest that let (X, d) be a complete metric space and T: X X be a mapping satisfying (d(Tx,Ty)) (d(x, y)) (d(x, y)) 7 for all x, y X, where,,,, : [0,) : [0,) are such that is an altering distance function, is continuous, is lower semi-continu Then T has a fixed point that is unique. Indeed, the contractive condition (1) may be expressed as: (d(Tx, Ty)) (d(x, y)) (d(x, y)), where : [0,) [0,) is provided by (t) = (t) + (t), t 0

Some theorems and their research problem and their future scope

1. Fixed Point Theorems in Fuzzy Metric Spaces

A brief introduction Authors Yonghong Shen, Dong Qiu, and WeiChen published this article in the journal Applied Mathematics in 2012. Several fixed point theorems for a novel class of self-maps in M-complete fuzzy metric spaces and compact fuzzy metric spaces were developed in this study. They utilised the definitions of t-Norm, M-Fuzzy Metric Space, and Complete Fuzzy Metric Space for this.

The Research Problem Theorem:

Let $(X,\ M,*)$ be an M-complete fuzzy metric space and T a self-map of X and suppose that ϕ : $[0,1] \rightarrow [0,1]$ satisfies the foregoing properties (P1) and (P2). Furthermore, let k be a function from $(0, \infty)$ into$(0,1)$. If for any $t > 0$, T satisfies the following condition: 4 $((Tx,\ Ty,t)) \leq k(t) \cdot \phi(M(x,\ y,t))$, Where $x,\ y \in X$ and$x \neq y$, then T has a unique fixed point. Whereϕ : $[0,1] \rightarrow [0,1]$which is used by altering the distance between two points satisfies the following properties: (P1) ϕ is strictly decreasing and left continuous; (P2) $\phi(\lambda) = 0$if and only if $\lambda = 1$. Obviously, we obtain that lim $\lambda \rightarrow 1$ $(\lambda) = (1) = 0$.

Future scope:

In this work authors worked on a new class of self-maps by altering the distance between two points in fuzzy metric space in which theϕ-functionwas used. On the basis of this kind of selfmap, authors proved some fixed point theorems in M- fuzzy metric space and complete fuzzy metric space. This property can also prove by changing another space.

2. Common Fixed Point Theorems For Commutating Mappings in Fuzzy Metric Spaces

Introduction This paper was published in 2012 Abstract and Applied Analysis by authors Famei Zheng and Xiuguo Lian. Authors generalized Jungck's theorem in fuzzy metric spaces and prove common fixed point theorems for commutative mappings in fuzzy metric spaces. For this purpose they have used the definition of t - norm , commutative map in fuzzy metric space.

The Research Problem Theorem:

Let $(X, M,*)$ be a complete fuzzy metric space and let $f : X \rightarrow X$ be a continuous map and $g : X \rightarrow X$ a map. If $(i) g(X) \subseteq f(X)$, (ii) g commutes with$f$, 5 (iii) $M(g(x), g(y),t) \geq M(f(x), f(y), \psi(t))$for all $x, y \in X$ and $t > 0$,where $\psi : [0, \rightarrow \infty) \rightarrow (0, \rightarrow \infty)$ is an increasing and left-continuous function $with\ \psi(t) > t$for all $t > 0$, then f and g have a unique common fixed point.

Future Prospects:

The authors of this research investigated a novel type of commutative mapping in fuzzy metric space using the left continuous function. Authors established certain fixed point theorems in fuzzy metric space using this type of commutative mapping. The partial order relation can also be used to generalise this trait. This research adds to our understanding of the fixed point theorem.

3.Notes on Fuzzy Metric Space Fixed Point Theorems

A brief introduction Authors Bijendra Singh and Mahendra Singh Bhadauriya submitted this research in the 2013 International Journal of Scientific and Engineering ResearchThe authors of this study used implicit relation to prove a fixed point theorem for A, B, S, T, P, and Q self maps. To get common fixed points, the concepts of weak compatibility and compatibility of type () of self maps are also applied.

The Research Challenge

Allow (X, M,) to be a full fuzzy metric space, and A, B, S, T, P, and Q to be mappings from X into itself that satisfy the following conditions:

(a) $P(X) \subset ST(X),Q(X) \subset AB(X)$, (b) Either $A\ B$ or P is continuous. (c) (P, AB) is compatible of type (β) and (Q, ST) is weaklycompatible. (d) $AB = BA$, $ST = TS$, $PB = BP$, $= TR$. (e) there exist (0,1) $\in k$ such that for every $x\ y \notin X$, and$t > 0$. $\emptyset(M(Px ,Qy, kt), M(ABx, STy,t), M(Px, ABx,t))$, $M(Qy, STy, kt)$, $M(Px, STy,t) \geq 0$. Then A, B, S, T, P and Q have a unique common fixed point in X.

Future Scope

In Fuzzy Metric Spaces, the authors constructed a single Fixed Point Theorem that satisfied implicit relations for weakly compatible mappings. Some possible applications include engineering, economics, and information systems in dealing with challenges resulting from approximation theory. In the future, researchers will discover new implicit relationships to relax situations.

CONCLUSION

Metric space is applicable to abilities and confidence in reading, interpreting, and writing mathematical reasoning are growing as you learn the terminology and definitions and apply the results.

You should now be able to do the following:

• Recognize the Euclidean distance

• function on Rn and comprehend its features, as well as state and apply the triangle and Reverse Triangle Inequality for the

• Euclidean distance function on Rn

• a particular distance function is a metric by explaining the geometric meaning of each of the metric space features.

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