THE STUDY OF LINEAR TRANSFORMATION

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Abstract

The place of this part is to give an audit of the examination that we have been driving in our exploration about the direct change idea, focusing in on inconveniences related with its learning, common mental models that understudies may make regarding it, a system of an inherited rot that depicts an expected way by which this idea can be created, issues that understudies may understanding as to registers of depiction, and the work that unique estimation conditions may play in interpreting its things. Preliminary results from a consistent assessment about picturing the pattern of a straight change are represented. A composing review that clearly relates to the substance of this part similarly as headings for future exploration and didactical proposition are given.

Keywords

Vector space, Binary compositions, Linear transformation, Rank – Nullity Theorem, The Algebra of Linear Transformations,Properties of linear transformation, Matrix representation**.**

Introduction

We have studied the concepts of binary compositions, relations and mapping. In this paper we shall study an algebraic system with a binary operation defined on its elements and satisfying some postulates (oraxioms). This algebraic system which occurs naturally in various mathematical situations (Vector space, linear transformation). Thestructure of this is one of the simplest mathematical structure. Now we shall consider important algebraic system known as **'Linear VectorSpace'**. The definition of vector space involves two sets, these sets of vectors V and the set of scalars F (which is always a field). We shall using two operations in the definition of a vector space, one is **internal composition** on the elements of V and the other is an **external composition** on the elements of V by the elements of F.

Linear transformation contains one vector space to another vector space with satisfying some conditions. In particular, a linear transformation from R^n to R^m is known as Euclidean linear transformation. In Linear Algebra, a transformation between two vector spaces is a rule that assigns a vector in a one space to a vector in the other space. These transformations are satisfied a certain property addition and scalar multiplication. This transformationrelates "image'' and "range'' and this makes these transformation different from other transformations.

Algebraic Structure

A set having one or more binary composition is called Algebraic Structure.

Types of compositions:

Some important types of binary compositions which will be used in defining algebraic structures such as Fields, Vector spaces.

1) **Commutative Composition:** A binary composition * on a set A is called commutative composition iff $x * y = y * x \; \forall x, y \in A$.
2) **Associative Composition:** A binary composition * on a set A is called associative composition iff $(x * y) * z = x * (y * z) \; \forall \; x, y, z \in A$.
3) **Composition with identity element:** A binary composition * on a set A is called a composition with identity element iff $\exists \; e \in A$, such that $e * x = x = x * e \; \forall \; x \in A$.
   This element e is named as identity element of A, which is always unique.
4) **Invertible Elements:** Let e be the identity element of set A under the composition '*' on the set A. Let $\alpha \in A$, then $\beta \in A$ is called an Inverse element of α iff $\alpha * \beta = e = \beta * \alpha$.
   Then the composition * is a composition with inverse element, which is always unique.
5) **Distributive Operations:** Let * and + be two binary operations on a set A, then we say that operation * is distributive with + if
   $x * (y + z) = (x * y) + (y * z) \; \forall \; x, y, z \in A$ (Left distributive law)
   $(y + z) * x = (y * x) + (z * x) \; \forall \; x, y, z \in A$ (Right distributive law)

Fields

A non – empty set F having at least two elements with two binaryCompositions "+" and "." (i.e. addition and multiplication) is called a field iff the following postulates (axioms) are satisfied.

Properties of Addition

i. $\forall \; a, b \in F \Rightarrow a + b \in F$ [Closure Property]
ii. $\forall \; a, b, c \in F \Rightarrow (a + b) + c = a + (b + c)$ [Associative Property]
iii. $\forall \; a \in F, \exists \; 0 \in F$ such that
   $a + 0 = a = 0 + a$ [Existence of Additive Identity]
iv. $\forall \; a \in F, \exists \; -a \in F$ such that
   $a + (- a) = 0 = (- a) + a$ [Existence of Additive inverse]

v. $\forall$ a, b $\in$ F $\Longrightarrow$ a + b = b + a [Commutative Property]

Properties of Multiplication

vi. $\forall$ a, b $\in$ F $\Rightarrow$ a b $\in$ F [Closure Property]

vii. $\forall$ a, b, c $\in$ F $\Rightarrow$ a (b c) = (a b) c [Associative Property]

viii. $\forall$ a $\in$ F, $\exists$ 1$\in$ F such that

1. a = a = a . 1

Here 1 is called unity of F.

ix. $a\ (\neq 0) \in$ F, $\exists$ b $\in$ F such that

a.b = b.a = 1

Here b is called inverse of a. [Existence of Multiplicative inverse]

x. $\forall$ x,y $\in$ F $\Longrightarrow$ x + y = y + x [Commutative Law]

Distributive Laws

$\forall$ x, y, z$\in$ F

x. (y + z) = x. y + x. z

(y + z). x = y. x + z. x

VectorSpace

Let (F, +,·) be a given field and V be a non – empty set with two compositions, one is internal binary composition on V, called addition of vectors and is denoted by + or $\oplus$ and the other external binary composition on V by the elements of F, called scalar multiplication and is denoted multiplicatively, then the given set V is called a **vector space orlinear space** over the field F iff the following axioms are satisfied.

Properties of Addition

1 Closure Property: $\forall$ x, y$\in$V we have x + y $\in$ V

2 Associative Property:$\forall$ x, y, z $\in$ V

We have (x + y) + z = x + (y + z).

3 Existence of Addative Identity:

There exists an element 0 $\in$ V

such that x + 0 = 0 + x = x $\forall$ x $\in$ V

Here 0 is known as zero vector in V or ad dative identity.

4 Existence of Addative Inverse:

For each element x ∈ V, there exists an element –x ∈ V such that

$x + (-x) = 0 = (-x) + x$

The element –x is called ad dative inverse of x.

5 Commutative Property:∀ x, y ∈ V

We have x + y = y + x.

Properties of Scalar Multiplication

M-1$\forall \alpha \in F$, x ∈ V we have α x ∈ V

M-2 $\forall \alpha, \beta \in F$, x ∈ V we have $(\alpha + \beta)\, x = \alpha\, x + \beta\, x$.

M-3 $\forall \alpha \in F$, x, y ∈ V we have $\alpha\, (x + y) = \alpha\, x + \alpha\, y$.

M-4 $\forall \alpha, \beta \in F$, x ∈ V we have $(\alpha\beta)\, x = \alpha\, (\beta\, x)$.

M-5 ∀ x∈ V, we have 1. x = x where 1 is the unity element of F.

Binary Compositions

Internal Composition: Let A be a set, then the mapping f: A × A → A is called internal composition on it.

External Composition: Let A and F be two non – empty sets. Then a mapping f: A × A → A is called external composition on A by the elements of F**.**

Linear Transformation

If V(F) and W(F) are two vector spaces, then a mapping T from V to W i.e.; T: V ⟶ W is said to be a **Linear Transformation** if and only if

i. T (v + w) = T (v) + T (w) for all v, w ∈ V.
ii. T (a v) = a T (v) for all v ∈ V and a ∈ F.

Properties of Linear Transformation

If T: V⟶ W is a linear transformation from V(F) to W(F). Then

i. T(0) = 0, where **0** on left hand ∈ V and **0** on right hand ∈ W.
ii. T(-x) = -T(x) where x ∈ V.

iii. $T(x - y) = T(x) – T(y)$ for all $x, y \in V$.
iv. $T(px) = pT(x)$ for all $x \in V, p \in I$.

Rank and Nullity of Linear Transformation

RANK: If V (F) and W (F) be a vector spaces and $T : V \rightarrow W$ be a L .T . , then the dimension of range space of T is called the rank of T and is denoted by ρ (T)

Thus ρ (T) = dim (Range T)

Nullity: If V (F) and W (F) be a vector spaces and $T : V \rightarrow W$ be a L .T . , then the dimension of null spaces of T is called the nullity of T and is denoted by ν (T)

Thus ν (T) = dim (Null space of T)

Range: If V (F) and W (F) are vectors spaces and $T: V \longrightarrow W$ is a Linear Transformation, then the image set of V under T is R (T) or T (V) i .e.,

Range T = {T (v) | v $\in$V}

Range T is also called Range Space. ($\because$ R (T) is a vector space)

Null Space or Kernel: If V (F) and W (F) are two vectors spaces and $T: V \longrightarrow W$ is a linear transformation then the set of all those vectors in V whose image under T is zero, is called Kernel or Null Space of T, which is denoted by N (T), i.e. , Null space of T = N (T) = {v $\in$V; T (v) = 0 $\in$ W}

RANK - NULLITY THEOREM OR SYLVESTER'S LAW OF NULLITY

If V (F) and W (F) are vector spaces and $T: V \rightarrow W$ is a linear transformation. Suppose V is a dimension of n. Then

$\therefore$ Rank T + Nullity T = n

$\Rightarrow$ Rank (T) + Nullity (T) = dim V

($\because$ V is finite dimensional)

$\Rightarrow$ Rank T + Nullity T = dim V

R (T) and N (T) are also finite dimensional.

THE ALGEBRA OF LINEAR TRANSFORMATIONS

Let U and V be vector spaces over the same field F and L (U , V) be the set of all linear transformations from V to W. L (U, V) is also a vector space over the same field and the set L (U, V) or Hom. (U , V) of all linear transformation form U (F) into V (F) is a vector space over the field F with addition scalar multiplication defined by

$(T_1 + T_2)(x) = T_1(x) + T_2(x) \ \forall\, x \in V$ and $T_1, T_2 \in L(U, V)$

$(\alpha T)(x) = \alpha T(x)$ for all $x \in V$ and $\alpha \in F$, $T \in L(U, V)$.

∴ L (U, V) is a vector space over F.

The set L (U, V) of all linear operators on V i.e. , linear transformations V into V forms a vector space with respect to addition and scalar multiplication.L (U, V) is defined only when U and Vare vector spaces over the same field.

In other way , if V (F) and U (F) are finite dimensional , then the vector space of all linear transformations from V to W is also finite dimensional and its dimension is equal to [dim. (U)] [dim. (V)].

∴ L (U, V) is a finite vector space having m n elements.

Thus dim. L (U, V) = m n = dim. U dim. V

If T: V ⟶ V is a linear operator, then L (V, V), the vector space of all linear operators on V is finite dimensional and dim $[L(V, V)] = (\dim V)^2$

MATRIX REPRESENTATION OF A LINEAR TRANSFORMATION RELATIVE TO BASIS

Let T: V ⟶ W be a linear transformation, where V and W are vector spaces over a field F and dim V = n and dim W = m

∵ T: V ⟶ W is a L.T. so that for every v ∈ V, we have T (v) ∈ W, so each

T (v) ∈ W can be uniquely written as a linear combination of the elements of B.

In particular, each $T(v_j) \in W$ where $1 \le j \le n$, can be expressed as:

i.e., T (v_j) $=\sum_{i=1}^{m} \alpha_{ij} w_i$, where α_{ij} ∈ F for 1 ≤ i ≤ m , 1 ≤ j ≤ n.

PARTICULAR CASE:

If W = V i.e. , T : V → V is a linear operator.We have the matrix of T w.r.t. , basis B as [T] = $[\alpha_{ij}]_{n \times n}$ i.e. , matrix have order n x n.

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Conclusion

In this paper we conclude that about visualization and understanding of linear transformation. There were various assumptions and rumors related to the concept of linear transformation which are conflicted in this paper leading to a better understanding and knowledge about linear transformation .Several Intuitive models related to this concept are also discussed in this paper. Various past discoveries and their merits and demerits are taken in consideration and the new concept introduced by scientists have also been taken in this paper.

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