A Novel Technique for Solving Integer Linear Bilevel Programming Problems

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ABSTRACT

A novel technique that addresses the solution of the general integer linear bilevel programming problem to global optimality is presented i.e. the general case of bilevel linear programming problems where each decision maker has objective functions conflicting with each other. We introduce linear programming problem of which resolution can permit to generate the whole feasible set of the upper level decisions. The approach is based on the relaxation of the feasible region by convex underestimation. Finally, we illustrate our approach with a numerical example.

Key words: *Bilevel Optimization, Integer linear bilevel programming, Stackelberg game*

1. Introduction

Bilevel programming problems occur in diverse applications, such as transportation, economics, ecology, engineering and others. They have been extensively studied in the literature [1-3]. However, when facing a real- world bilevel decision problem, the leader and the follower may have multiple conflict objectives that should be optimized simultaneously for achieving a solution [4]. There are only very few approaches in the literature dealing with bilevel multiobjective problems: less than a dozens of paper in the literature are related to this particular class of problems to our knowledge [5-8]. Three reasons at least can explain the fact that the issue has not yet received a broad attention in the literature: the difficulty of searching and defining optimal solutions; the lower level optimization problem has a number of trade off optimal solutions; and it is computationally more complex than the conventional Multiobjective Programming Problem or a bilevel Programming Problem. Consequently, it is extremely desirable to develop a simple and practical technique that can permit to find efficient solutions for this class of bilevel programming problem.

The paper is organized as follows. In the next section, we recall some notions about the optimistic formulation of BPP. In Section 3, steps for solve linear bilevel programming problem (LBPP). Section 4 presents. Section 5 presents A Novel Technique for Solving Linear Bilevel Programming Problems Finally, the paper is concluded in Section 6.

2. Optimistic Formulation of BPP

A Bilevel Programming Problem (BPP) is a decision problem where the vector variables x and y are controlled by two decision-makers: the leader and the follower. Variables x (resp. y) are variables of decision at the upper (resp. lower) level. This structure of hierarchical optimization appears in many applications when the strategic y of the lower level depends on the strategic x of the upper level. A standard Bilevel Programming Problem (BPP) can be modeled as follows:

$$\min_x F(x,y)$$
$$\text{subject to} \begin{cases} G(x) \leq 0 \\ y \text{ solve} \begin{cases} \min f(x,y) & (BPP) \\ s.t & \\ g(x,y) \leq 0 \end{cases} \end{cases}$$

Mathematically, solving a BPP consists of finding a solution of the problem at the upper level called the leader's (or ouster's) problem. where for each value of x, y is the solution of the problem at the lower level, which is called the follower's (or inner's) problem; with $x \in \mathbb{R}^{n1}$, $y \in \mathbb{R}^{n2}$; F, f : $\mathbb{R}^{n1+n2} \rightarrow \mathbb{R}^{m1}$ are the objective functions of the upper (resp. lower) level; G, g : $\mathbb{R}^{n1+n2} \rightarrow \mathbb{R}^{m2}$ are the constraint functions of the upper (resp. lower) level.

In the literature, the BPP and the problem with multiple objectives at the upper level or at the lower level are presented as a class of bilevel problems and are at the center of research of some authors such as [9, 10]. Fatehem et al. [11] present Particle Swarm Optimization (PSO) algorithm for solving the bilevel programming problem with multiple linear objectives at the lower level while supposing the objective function at the upper level quasiconcave. They conclude that the feasible region of the problem consists of faces of the polyhedron defined by the constraints. O. Pieume, L. P. Fotso et al. [12, 13] study Bilevel Multiobjective Programming Problem (BMPP). For the linear case, they establish equivalence between the feasible set of a bilevel multiobjective linear programming and the set of efficient points of an artificial multiobjective linear programming problem. The same authors [12] show how to construct two artificial multiobjective programming problems such that any point that is efficient for both problems is an efficient solution of a BMPP.

A standard Integer Linear Bilevel Programming Problem (ILBPP) can be modeled as follows:

$$min_x \, F\,(x, y) = c_1 x + d_1 y$$

$$\text{subject to} \begin{cases} A_1 x + B_1 y \le b_1 \\ y \text{ solve} \begin{cases} \min f\,(x, y) = c_2 x + d_2 y \\ s.t \\ A_2 x + B_2 y \le b_2 \end{cases} \end{cases}$$

3. Steps for Solve Linear Bilevel Programming Problem (LBPP)

Step1: Constraint region of the BLPP:

$$S = \{(x, y) : x \in X, y \in Y, A_1 x + B_1 y \le b_1, A_2 x + B_2 y \le b_2\}$$

Step2: Follower's feasible set for each fixed $x \in X$:

$$S(x) = \{y \in Y : B_2 y \le b_2 - A_2 x\}$$

Step3: Follower's rational reaction set:

$$P(x) = \{y \in Y: y \in \text{argmin}\,[f\,(x, \acute{y}): \acute{y} \in S(x)]\}$$

Step4: Inducible Region:

$$IR = \{(x, y) \in S, y \in P(x)\}$$

Step5: When S and P(x) are non-empty, the BLPP can be written as:

$$\min\,\{F\,(x, y): (x, y) \in IR\}$$

4. A Novel Technique for Solving Linear Bilevel Programming Problems

Step1: Constraint region of the BLPP:

$$S = \{(x, y) : x \in X, y \in Y, A_1 x + B_1 y \leq b_1, A_2 x + B_2 y \leq b_2\}$$

Step2: Follower's feasible set for each fixed x ∈ X:

$$S(x) = \{y \in Y : B_2 y \leq b_2 - A_2 x\}$$

Step3: Follower's rational reaction set:

$$P(x) = \{y \in Y: y \in \text{argmin}\ [f(x, \acute{y}): \acute{y} \in S(x)]\}$$

Step4: Inducible Region:

$$IR = \{S(x) \cap P(x)\}$$

Step5: When S and P(x) are non-empty, the BLPP can be written as:

$$\min \{F(x, y): (x, y) \in IR\}$$

5. Illustrative Example

- Example 1 is taken from [14]. Let consider the leader's problem and the follower's problem:

$$min_x\, F(x, y) = x - 4y$$

$$\textbf{s.t} \begin{cases} \min f(y) = y \\ \boldsymbol{s.t} \begin{cases} -x - y \leq -3 \\ -2x + y \leq 0 \\ 2x + y \leq 12 \\ -3x + 2y \leq -4 \end{cases} \\ \boldsymbol{x, y} \geq 0 \end{cases}$$

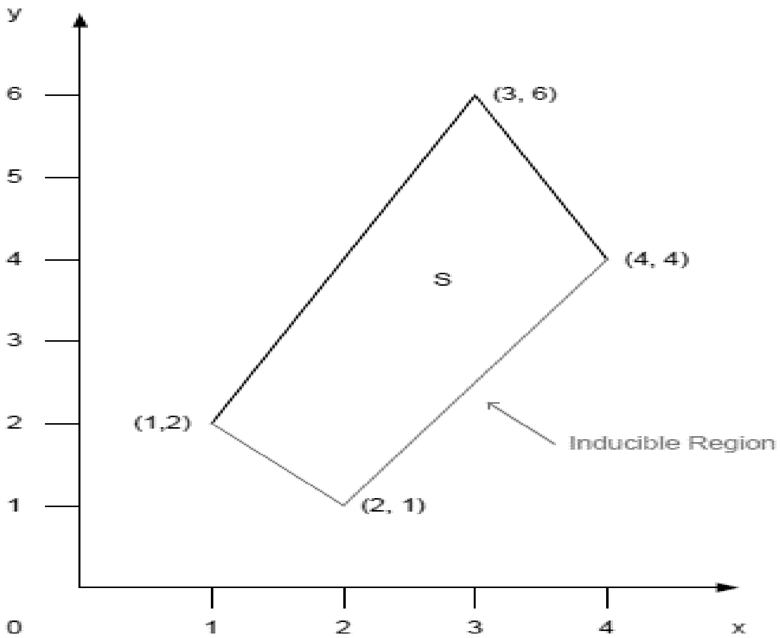

Fig 1: feasible region S for leader and follower

x*	y*	F*	f*
4	4	-12	4

My technique: As Fig 1

S(x) = {1, 2, 3, 4}

P(x) = {1, 2, 4}

IR = {S(x) ∩ P(x)}= {(1,2), (2,1), (4,4)}

Point	x*	y*	F*	f*
(1,2)	1	2	-7	2
(2,1)	2	1	-2	1
(4,4)	**4**	**4**	**-12**	**4**

The solution is the same of [14].

- Example 2 is taken from [15]. Let consider the leader's problem and the follower's problem:

$$min_x \, F\,(x, y) = -x - 2y$$

$$\textbf{s.t} \begin{cases} 2x - 3y \geq -12 \\ x + y \leq 14 \\ \min f\,(y) = -y \\ \boldsymbol{s.t} \begin{cases} -3x + y \leq -3 \\ 3x + y \leq 30 \end{cases} \\ \boldsymbol{x}, \boldsymbol{y} \geq 0 \end{cases}$$

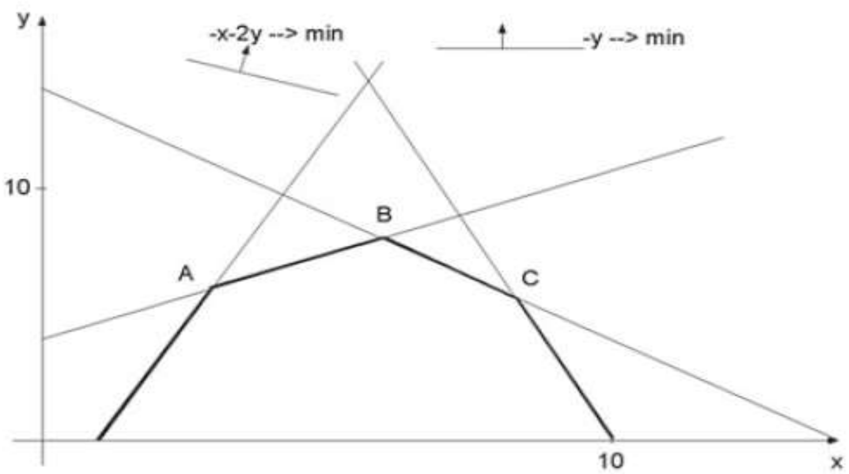

Fig 2: feasible region S for leader and follower

x*	y*	F*	f*
6	8	-22	-8

My technique: As Fig 2 the final solution

Point	x*	y*	F*	f*
(1,0)	1	0	-1	0
(6,8)	**6**	**8**	**-22**	**-8**
(8,6)	8	6	-20	-6
(10, 0)	10	0	-10	0

The solution is the same of [15].

6. Conclusion

In this paper, we have uniquely defined a lower level solution for every upper level feasible solution as a parameter of the follower's problem. We have formulated a bilevel programming Problem (BPP), of which we have considered the objective function and showed that there was an extreme point of the feasible space that was an optimal solution of the BPP. An upper bound to the global minimum is obtained by transforming the original problem into a single level one without the relaxation and solving for local optimality. After upper and lower bounds are obtained to the global solution, the initial region of the problem variables is partitioned into smaller regions by using one of the branching rules that are developed within the deterministic global optimization algorithm. Several examples of varying features are presented to show the capability of the approach in solving various ILBPP problems.

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