

A Study of Modified Newton-Raphson Method

Anushka Chauhan

Department of Mathematics, Chandigarh University, India

E-mail ID: anushkachauhan33@gmail.com

Abstract: A basic alteration of the standard Newton technique is investigated and described for the approximation of the roots of a univariate function. For a similar number of functions and evaluation of the derivative, an altered strategy combines quicker, with the convergence of the modified NR's method being 2.4 as compared with the regular NR method which is 2. Some of the example shows the faster convergence accomplished with the modified NR method. This modification of Newton's technique is generally basic and strong. It is bound to converge to the solution rather than the higher order or Newton-Raphson method itself. In this paper, the modification of NR strategy introduced which offers expanded rate of convergence over NR standard method.

Keywords: Newton's method, modified Newton-Raphson method, convergence, iterative method.

INTRODUCTION

Using numerical methods for finding the roots of an equation is, undoubtedly, most interesting problems of applied math.[1] For a long time, the issue of finding mathematical solutions of non-linear equations has been an extremely dynamic field. The roots of an equation are associated with the convergence of the iterative method.

Newton-Raphson technique for finding the root of a non-linear equation i.e. $f(t)=0$ has for quite some time been supported for its fast convergence. By just using its first derivative and function, Newton's technique produces an arrangement of approximation iteratively that converges to a simple root quadratically.

The standard NR method, iteratively approximate the zero of a function $f(t)$ by using the tangent of the curve, while the modified NR method [8] uses $f(t)$ and $p(t) = f(t)/f'(t)$ have same zeros but the convergence properties of $p(t)$ is better, given as

$$p(t) = \frac{f(t)}{f'(t)}$$

The iteration of modified Newton-Raphson method for $p(t)$,

$$t_{m+1} = t_m - \frac{p(t_m)}{p'(t_m)}$$

$$p'(t) = \frac{[f'(t)]^2 - f(t)f''(t)}{[f'(t)]^2}$$

$$t_{m+1} = t_m - \frac{[f'(t_m)]^2}{[f'(t_m)]^2 - f(t_m)f''(t_m)} \frac{f(t_m)}{f'(t_m)}$$

$$t_{m+1} = t_m - \frac{f(t_m)f'(t_m)}{[f'(t_m)]^2 - f(t_m)f''(t_m)}$$

Newton-Raphson method produces a sequence of approximations that converges quadratically even for multiple roots.

Let t_r be non-repeated roots of $p(t) = 0$ and $f(t)$ has k roots which are repeated such that, $m \geq 2$

$$f(t) = h(t) \cdot (t - t_r)^k$$

where $h(t_r) \neq 0$, then

$$f'(t) = h'(t)(t - t_r)^k + h(t)k(t - t_r)^{k-1}$$

$$f'(t) = (t - t_r)^{k-1}[h'(t)(t - t_r) + kh(t)]$$

$$\text{and} \quad p(t) = \frac{f(t)}{f'(t)}$$

$$f'(t) = \frac{h(t) \cdot (t - t_r)^k}{(t - t_r)^{k-1}[h'(t)(t - t_r) + kh(t)]}$$

$$f'(t) = \frac{h(t)(t - t_r)}{h'(t)(t - t_r) + kh(t)}$$

Hence, t_r is non-repeated root of $p(t) = 0$.

LITERATURE REVIEW

Various authors have inferred multistep, predictor corrector methods which offers convergence of higher order however require just its derivative and the function.

Weerakoon and Fernando [3] showed in **2000** that VNM (Variant of Newton's Method) is convergent of at least third order required that there exist the 1st, 2nd and 3rd derivatives of the function. The main attribute of VNM was it isn't needed to do iteration to calculate 2nd or higher derivatives. However, as compared with NR method, VNM requires one extra evaluation of the function in each iteration.

In 2003, H. Homeiera [4] considered a modification of NR technique for finding the zero of the univariate function and demonstrated that cubically this modification converges. In every iteration, it needs one function evaluation and two derivative evaluations. The modification is appropriate if the estimation of the derivative suffers a lower or similar cost than that of the actual function.

In 2007, Jisheng Kou [5] has obtained numerous new improvements of NR technique. He demonstrated that the strategies have 5 or 6 order of convergence. Examination of effectiveness claims that these techniques may cope with NR strategy, that is likewise exhibited by mathematical consequences. Likewise, dependent on these techniques, he built some new multi-step iterative processes that have higher convergence than multistep NR strategy.

In 2011, P. Wang [6] introduced another group of modification of NR-like techniques which incorporates, as two specific cases, the trapezoidal and midpoint rule. The techniques look for, one function evaluation and two 1st order derivative evaluation. He showed that every family yields a convergent solution cubically, and seen from mathematical examples that the proposed strategies show similar execution as that of other known techniques for a similar order.

In 2012, Soleymani et al. [7] examined the development of few two-step iteration processes for discovering straightforward roots of non-linear scalar conditions without iterative classes of strategies. The classes are worked through the methodology of weight function and these acquired classes come to the 4th order with the help of one function and two 1st evaluation of the derivative for every full iteration which shows that Jarratt type plans can be considered by classes.

These strategies all depend upon evaluation of function or evaluation of derivative in each iteration than NR strategy, however this extra expense is counterbalanced by the convergence of higher order.

THE MODIFICATION OF N-R'S METHOD

When the iteration is set up, overall iteration takes the form given below,

$$t_m^* = t_m - \frac{f(t_m)}{f' \left(\frac{t_{m-1}}{2} + \frac{t_{m-1}^*}{2} \right)}$$

$$t_{m+1} = t_m - \frac{f(t_m)}{f' \left(\frac{t_m}{2} + \frac{t_m^*}{2} \right)}$$

The above equation is the predictor-corrector method where the predictor iteration step depends upon the derivative determined in previous iterative step and the corrector iteration is inspired by connection implied which is

$$t_r = t_0 - \frac{f'(t_0)}{f' \left(\frac{t_0}{2} + \frac{t_r}{2} \right)}$$

The uncommon component of the plan is the interleaving of evaluation of function and derivative at various values of t .

Overall modification of NR strategy that we analyze is given by $t_0^* = t_0$

$$t_1 = t_0 - \frac{f'(t_0)}{f' \left(\frac{t_0}{2} + \frac{t_0^*}{2} \right)} = t_0 - \frac{f(t_0)}{f'(t_0)}$$

Now for $m \geq 1$

$$t_m^* = t_m - \frac{f(t_m)}{f' \left(\frac{t_{m-1}}{2} + \frac{t_{m-1}^*}{2} \right)} \quad (i)$$

$$t_{m+1} = t_m - \frac{f(t_m)}{f' \left(\frac{t_m}{2} + \frac{t_m^*}{2} \right)} \quad (ii)$$

These procedures in our modification of NR methodology [1] are delineated in the figure given below, where the steps are appeared at finding t_2 .

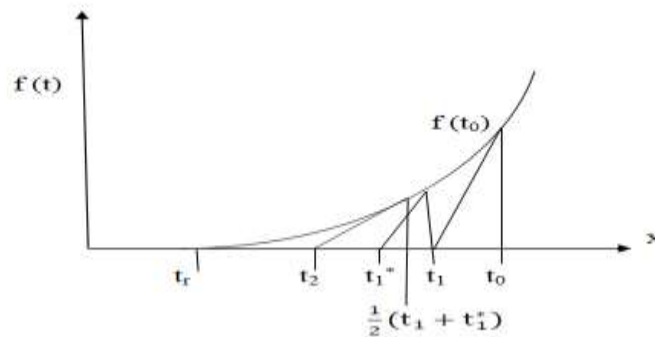


FIGURE 1. NR METHODOLOGY

The highlights of modified NR technique can be found in this, in particular

1. The determination of the value from t_1 utilizing $f(t_1)$ and the value of the evaluation of the derivative at

$$\frac{1}{2}(t_1 + t_1^*)$$

2. In the following predictor step to get t_3^* we rewrite the derivative value.

This re-utilization of derivative implies that starred values evaluation of t in (i) basically come free, which at that point empowers the more proper derivative value to be utilized in the corrector step (ii).

COMPARISONS OF VARIOUS ITERATIVE TECHNIQUES

- **Newton-type methods:** -Many authors have introduced mathematical schemes which have cubic convergence.[3] In every cycle of mathematical schemes 3 function evaluation or derivative evaluation are expected. The most ideal method of comparing these mathematical plans is to set per evaluation of function or evaluation of the derivative the rate of convergence, the purported efficiency of the mathematical plan. Therregular NR technique has an efficiency of $1.4142 (2^{1/2})$, $1.5538 [(\sqrt{2}+1)^{1/2}]$ is the efficiency of the modified NR technique, while $1.4422 (3^{1/3})$ is the efficiency of cubic convergence strategies.
- **Secant method:** -These efficiency of secant method which is $1.6180 [0.5(1+\sqrt{5})]$ is also be compared with these efficiencies. This efficiency is larger than modified NR technique. The drawback of secant

method was that the small commotion could influence the denominator and the stopping criteria won't generally stay away from this undesirable behaviour.

- **Line search method:** - As an improvement of the NR strategy [2] the line-search method primarily fills in as a remuneration of initial estimate. In every iteration m , the line-search calculation attempts to streamline a linear, quadratic, or cubic approximation of f along a plausible search bearing j_m .

$$t_{m+1} = t_m + \beta_m j_m, \quad \beta_m > 0$$

By figuring a roughly ideal scalar

$$|f(t_m + \beta_m j_m)| < |f(t_m)|$$

In methodology, an appropriate decision can ensure the inequality:

$$|f(t_{m+1})| < |f(t_m)|$$

Hence, modified Newton Raphson method is significantly better than line search method.

- **Fourth order iterative method:** - Fourth order scheme was built up by Jarratt [7] that requires just 1 evaluation of the function and 2 evaluations of the derivative and comparative fourth order plans have been portrayed by Soleymani, Khattri and Vanani. Jarratt's plan is like those of Jisheng Kou in that if at predictor and corrector iteration steps the proportion of the derivatives surpasses a factor of 3, the strategy gives a limitless change in t .
- **Modified Halley's method:** - Kou [5] has built up a few techniques that require 2 evaluation of the function and derivative and these strategies accomplish either 5 or 6 order of convergence, which have efficiencies of 1.4953 ($5^{1/4}$) and 1.5651 ($6^{1/4}$). The bigger of these two efficiencies is bigger than that of modified NR method by 1%. In these techniques, at various values t the denominator is a combination of derivatives that are assessed, with the goal that when the beginning value of t isn't near the root, this denominator can go to nothing & strategies will not converge. Of the four sixth order techniques, if the proportion of the derivative of the functions at two values of t varies by a factor of more than 3, at that point the strategy gives a boundless change in t .

CONCLUSION

In this paper, the modification of NR strategy introduced which offers expanded rate of convergence over NR standard method, practically speaking the adjusted strategy is found to offer more prominent effectiveness as far as absolute evaluation of the function than other purported cubic convergence strategies. It is the re-utilization of recently processed derivative values that gives modified technique its mathematical efficiency is contrasted with regular NR strategy.

Modification of NR technique is generally straightforward and is strong. It is bound to merge to an answer than are either the higher order plans or NR strategy itself. An extra benefit i.e. stopping criteria can be applied to changed NR technique after every either function evaluation or its derivative evaluation in computational effectiveness compared with strategies that require a few calculation of the function or derivative to finish a full iteration.

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