Hydrodynamic Forces on a Submerged Circular Cylinder in Two-layer Fluid with an Ice-cover

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Abstract

We consider problems based on linear water wave theory concerning the interaction of wave with horizontal circular cylinder submerged in two-layer ocean consisting of a upper layer of finite depth bounded above by an ice-cover and below by an infinite layer of fluid of greater density, the ice-cover being modelled as an elastic plate of very small thickness. Using the method of multipoles, we formulate the problems of hydrodynamic forces on a submerged cylinder in either the upper or the lower layer. The vertical and horizontal forces on the circular cylinder are obtained and depicted graphically against the wave number for various values of flexural rigidity of ice-cover to show the effect of the presence of ice-cover on these quantities. Also when the flexural rigidity and surface density of the ice-cover are taken to be zero, the ice-cover tends to a free-surface. Then all the forces are the same as in the case of two-layer fluid with free surface.

Keywords: Two-layer fluid, Water wave scattering, Submerged circular cylinder, Ice-cover, Vertical and horizontal forces.

1. Introduction

The problem of scattering and radiation problems for a submerged as well as floating bodies in water have been explored for decades for various shapes of bodies. Long back Dean [1], Ursell [2] investigated the problem of water wave scattering by a horizontal circular cylinder fully submerged in deep water using the method of multipoles. This method also used in various field of theoretical physics (cf. Jackson [3], Morse and Feshbach [4]). Evans and Linton [5] considered both the scattering and radiation water wave problems by a submerged circular cylinder in finite depth water as part of a need to determine accurately the natural frequencies of oscillation of a highly buoyant tethered cylinder. Also, one of the attractive features of multipole expansion method for this type of problem is that the evaluation of hydrodynamic forces on a submerged circular cylinder in water is investigated (cf. Linton and McIver [6], Taylor and Hu [7], Sturova [8]).

The effect of the density changes of sea water are usually ignored in the early studies and assumption of uniform density was often used for the fluid. Recently, the effects of density inhomogeneity have attracted more and more attention. Wave propagation in a two-layer fluid with a free surface and an interface was first investigated by Stokes [9]. For such a two-layer fluid, It is known that for a particular frequency, there exist two modes

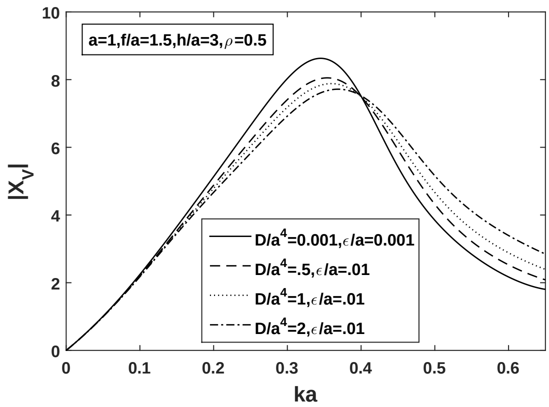

Fig 6. Vertical forces due to an incident wave of wave number λ_1 in upper layer

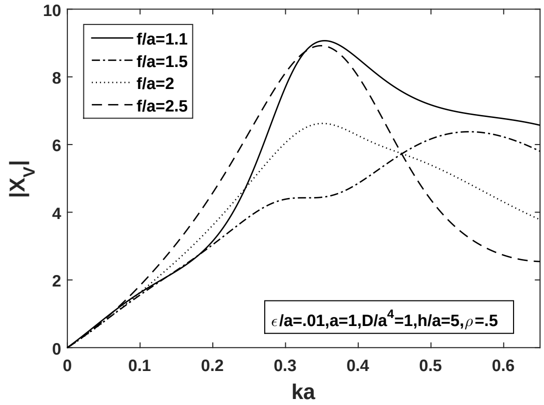

Fig 7. Vertical forces due to an incident wave of wave number λ_1 in upper layer

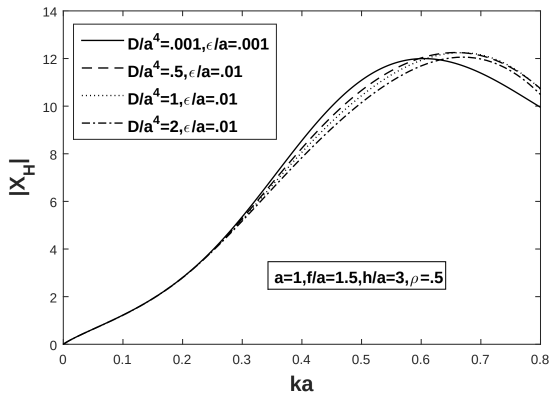

Fig 8. Horizontal forces due to an incident wave of wave number λ_1 in upper layer

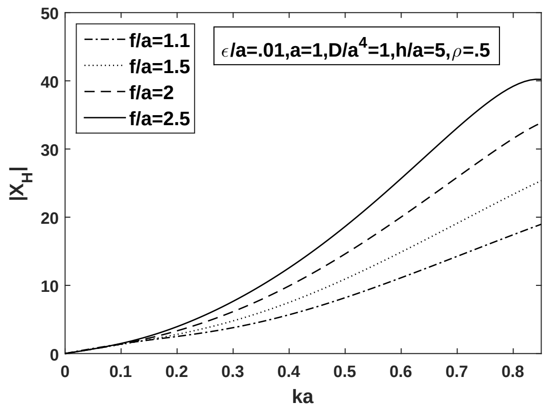

Fig 9. Horizontal forces due to an incident wave of wave number λ_1 in upper layer

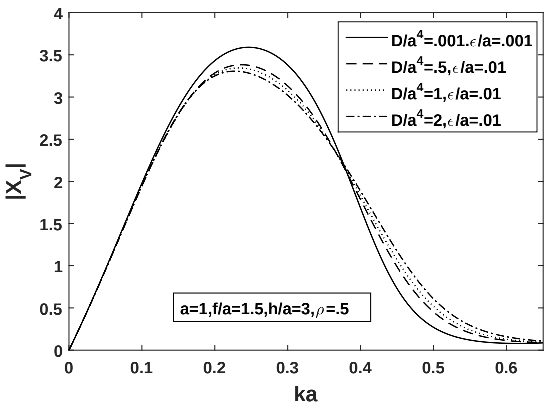

Fig 10. Vertical forces due to an incident wave of wave number λ_2 in upper layer

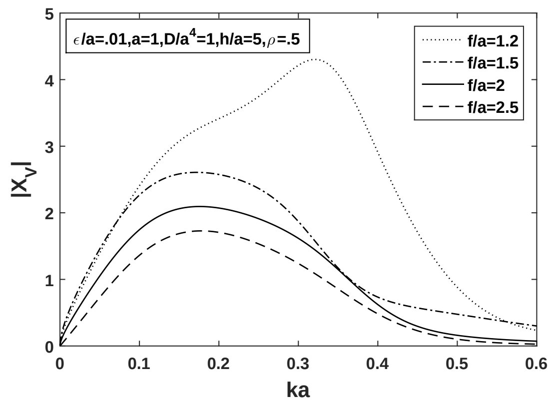

Fig 11. Vertical forces due to an incident wave of wave number λ_2 in upper layer

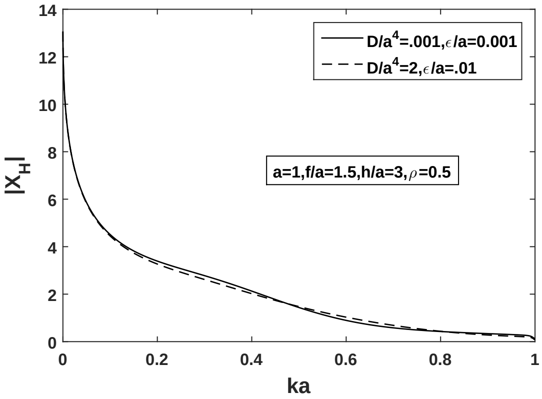

Fig 12. Horizontal forces due to an incident wave of wave number λ_2 in upper layer

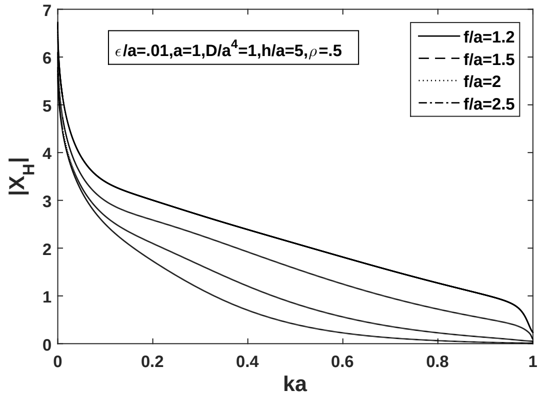

Fig 13. Horizontal forces due to an incident wave of wave number λ_2 in upper layer

Fig.8 and 12 show the horizontal forces for different values of D/a^4 due to an incident wave of wave number λ_1 and λ_2 respectively. These show the effect of the flexural rigidity of the ice-sheet and also observed that when $D/a^4 = .001$, $\epsilon/a = .001$, the forces are similar to the case of free surface two-layer fluid. Fig.9 and 13 show the horizontal forces for different values of f/a. For both the cases, it is observed that the forces increase with the increase of f/a. Also it is noted that from the figures the forces in the lower layer are smaller than that for upper layer for any value of D/a^4.

4. Conclusion

The problem of hydrodynamic forces on the submerged circular cylinder is extended here for a two-layer fluid. The lower is of infinite depth and the upper layer is of finite depth with an ice-cover surface, modelled as a thin elastic plate. Numerical results for the vertical and the horizontal forces on the cylinder are obtained. The method of multipoles has been shown to be an extremely powerful method for solving this problem involving submerged circular cylinder. The vertical forces on the cylinder in lower layer and also vertical and horizontal forces on the cylinder in upper layer are depicted graphically against the wave number in a number of figures. When $D/a^4 = 0.001$ and $\epsilon/a = 0.001$, the forces are coincided with the curves for the case of two-layer fluid with free surface.

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