Application of Picture Fuzzy Soft Relations in Multi- altribute Decision Making

P.Jayaraman *
Assistant Professor
Department of Applied Mathematics,
Bharathiar University,
Coimbatore - 641 046, India.

K.Vetrikkani †
Research Scholar
Department of Applied Mathematics,
Bharathiar University,
Coimbatore - 641 046, India.

A.Selvakumar ‡
Research Scholar
Department of Applied Mathematics,
Bharathiar University,
Coimbatore - 641 046, India.

R.Nagarajan §
Associate professor
J.J. college of Engineearing & Technology
Department of Mathematics,
Trichrappalli 620009, India.

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*E.mail: jrmsathya@gmail.com. (P.Jayaraman)
†E.mail: kathamuthuvetri@gmail.com.(K.Vetrikkani)
‡E.mail: selvabed@gmail.com.(A.Selvakumar)
§E.mail: rajenagarajan1970@gmail.com.(R.Nagarajan)

Abstract : In this paper, we first define picture fuzzy soft sets (PFSS) and study some of their relevant operations such as subset,equal, complement, AND,OR... and so on.we investigate some theorems on picture fuzzy soft sets based on union and intersection with counter examples. Also we proved a necessary and sufficent condition for the dual laws of PFSS theory.Finaly, we then introduce an algorithum based on relational picture fuzzy soft matrix to solve decision making problems.

Key words: fuzzy set, soft set,fuzzy softset,picture fuzzy soft set,subset,equal, AND,OR,complement,dual law,decision making.

1 Introduction

A fuzzy set was first introduced by Zadeh [23] and then the fuzzy sets have been used in there consideration of classical mathematics. Yuan.et.al.[22] introduced the concept of fuzzy subgroup with thresholds. A fuzzy subgroup with thresholds λ and μ is also called a (λ, μ)-fuzzy subgroup. A.Solairaju and R.Nagarajan introduced the concept of structures of Q- fuzzy groups [19]. A.Solairaju and R. Nagarajan studied some structural properties of upper Q-fuzzy index order, with upper Q- fuzzy subgroups[20]. Such inaccuracies are associated with the membership function that belongs to [0,1]. Through membership function, we obtain information which makes possible for us to reach the conclusion. The fuzzy set theory becomes a strong area of making observations in different areas like medical science, social sciences, engineering, management sciences, artificial intelligence, robotics, computer networks, decision making and so on. Due to unassociated sorts of unpredictability occurring in different areas of life like economics, engineering, medical sciences, management sciences, psychology, sociology, decision making and fuzzy set as noted and often effective mathematical instruments have been offered to make, be moving in and grip those unpredictability. Since the establishment of fuzzy set, several extensions have been made such as Atanassov's([3], [4], [5]. [6])work on

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intuitionistic fuzzy set (IFSs) was quite remarkable as he extended the concept of FSs by assigning non-membership degree say "N(x)" along with membership degree say "P(x)" with condition that $0 \leq P(x) + N(x) \geq 1$. Strengthening the concept IFS suggest pythagorean fuzzy sets which somehow enlarge the space of positive membership and negative membership by introducing some new condition that $0 \leq P^2(x) + N^2(x) \geq 1$. Molodtsov [14] introduced the concept of soft sets that can be seen as a newmathematical theory for dealing with uncertainty. The soft set theory has been applied to many different fields with great success. Maji.et.al. ([8],[9][10]) worked on theoretical study of soft sets in detail, and presented an application of soft set in the decision making problem using the reduction of rough sets. N -picture fuzzy soft sets studied in [12,13].Recently, Cuong [7] proposed picture fuzzy set (PFS) and investigated the some basic operations and properties of PFS. The picture fuzzy set is characterized by three functions expressing the degree of membership, the degree of neutral membership and the degree of non-membership. The only constraint is that the sum of the three degrees must not exceed 1. Basically, PFS based models can be applied to situations requiring human opinions involving more answers of types: yes, abstain, no, refusal, which cant be accurately expressed in the traditional FS and IFS. Until now, some progress has been made in the research of the PFS theory. Singh [17] investigated the correlation coefficients for picture fuzzy set and apply the correlation coefficient to clustering analysis with picture fuzzy information. Son [18] introduce several novel fuzzy clustering algorithms on the basis of picture fuzzy sets and applications to time series forecasting and weather forecasting. In this paper,we Define picture fuzzy soft sets (PFSS) and study some of their relevant operations such as subset,equal, complement, AND,OR... and so on.we investigate some theorems on picture fuzzy soft sets based on union and intersection with counter examples. Also we proved a necessary and sufficent condition for the dual laws of PFSS theory.Finaly, we then introduce an algorithum based on relational picture fuzzy soft matrix to solve decision making problems.

2 Preliminaries and Basic Concepts

2.1 Definition

Let the universal set be $R \neq \phi$. Then $A = \{< r, P_{A(r)} > / r \in R\}$ is said to be a fuzzy set of R,where $P_A : U \to [0,1]$. is said to be the membership degree of r in R.

2.2 Example

Let $R = \{r_1, r_2, r_3, r_4, r_5\}$ be the reference set of students. Let $\widetilde{A}$ be the fuzzy set of " smart students" where 'smart' is fuzzy term.$\widetilde{A} = \{< r_1, 0.1 >, < r_2, 0.4 >, < r_3, 0.7 >, < r_4, 1 >, < r_5, 0.9 >\}$. Here $\widetilde{A}$ indicates that the smartness of r_1 is 0.1 and so on.

2.3 Definition

A pair (δ, A) is called soft set over R, where F is a mapping given by $F : A \to P(R)$.

2.4 Example

Suppose $R = \{h_1, h_2, h_3, h_4, h_5, h_6\}$ be the the set of six houses and E={Expensive(e_1), Beautiful(e_2),Wooden(e_3),Cheap(e_4)} then the soft set (δ, E) is
$(\delta, E) = \{\{h_2, h_4\}\{h_1, h_3\}, \{h_3, h_4, h_5\}, \{h_1, h_3, h_5\}\}$ where each approximation has two parts

(i) a predicate,P;

(ii) an approximate value set V.

R	$Expensive(e_1)$	$Beautiful(e_2)$	$Wooden(e_3)$	$Cheap(e_4)$
h_1	0	1	0	1
h_2	1	0	0	0
h_3	0	1	1	1
h_4	1	0	1	0
h_5	0	0	1	1
h_6	0	0	0	0

thus, a sof set $(\delta, E) = \{P_1 = V_1, P_2 = V_2, P_3 = V_3, ...P_n = V_n\}$.

2.5 Definition

Let I^R denote the set of all fuzzy sets on X and $A \subset E$. A pair (δ, A) is called fuzzy soft set over R, where $\delta : A \to I^R$. that is, for each $a \in A$, $\delta_a :\to I^R$ is the fuzzy set on R.

2.6 Example

Let $R = \{r_1, r_2, r_3, r_4, r_5\}$ be a universial set and $E = \{e_1, e_2, e_3, e_4, e_5\}$ be a set of parameters.If $A = \{e_1, e_2, e_4\} \subseteq E$. $P_A(e_1) = \{0.2/r_2, 0.3/r_4, \}$, $P_A(e_2) = R$ $P_A(e_4) = \{0.5/r_1, 0.7/r_3, 0.9/r_5\}$ then the fuzzy soft set δ_A is defined as $\delta_A = \{r_1, (0.2/r_2, 0.3/r_4), (r_2, U), (r_4, (0.5/r_1, 0.7/r_3, 0.9/r_5)\}$.

2.7 Definition

Let the universe set be $R \neq \phi$. Then the set $A = \{< r,\ P_{A(r)},\ I_{A(r)},\ N_{A(r)} > /r \in R\}$ is said to be a picture fuzzy set of R,where $P_A : R \longrightarrow [0,1]$, $I_A : R \longrightarrow [0,1]$, $N_A : R \longrightarrow [0,1]$. are said to be the degree of r in R and the positive membership degree of r in R,and the neutral membership degree of r in R,and the nagative membership degree of r in R respectively. Also $P_{(A)}, I_{(A)}, N_{(A)}$ satisfy the following condition:$(\forall r \in R)$ $(0 \leq P_{A(r)} + I_{A(r)} + N_{A(r)} < 1)$. Then for $r \in R$, $\pi_{A(r)} = 1 - \{P_{A(r)} + I_{A(r)} + N_{A(r)}\}$ could be called the degree of refusal membership of r in R. Clearly, if $(r \in R)$, $\pi_{A(r)} = 0$, then A will be generated to be a standared intuitionistic fuzzy set . If $(\forall r \in R), I_{A(r)} = 0$ and $\Pi_{A(r)} = 0$,then A will be generated to be a classical fuzzy set . Let $\Pi_{A(r)}$ denote the set of all fuzzy sets of R. For the sake of simplicity picture fuzzy set is denoted by PFS.

2.8 Example

Basically, the model picture fuzzy set may be adequate in situations when we face human opinions involving more answers of type:(i) Yes (ii)abstain (iii) No (iv) Refusal.Voating can be a good example of such a situation as the human voters may be divided into four groups of those who:(i) Vote for (ii) abstain (iii) Vote aganist (iv) Refusal of the voting.

2.9 Definition

For $A, B \in PF(R)$, define (i)$A \subseteq B \iff P_{A(r)} \leq P_{B(r)},\ I_{A(r)} \leq I_{B(r)} and\ N_{A(r)} \leq N_{B(r)}$.
(ii) $A = B \iff A \subseteq B$ and $B \subseteq A$.
(iii)$A \cup B = \{(r, max\{(P_{A(r)}, P_{B(r)})\},\ min\{(I_{A(r)}, I_{B(r)})\}, min\{(N_{A(r)}, N_{B(r)})\}/r \in R\}$.
(iv)$A \cap B = \{(r, min\{(P_{A(r)}, P_{B(r)})\},\ min\{(I_{A(r)}, I_{B(r)})\}, max\{(N_{A(r)}, N_{B(r)})\}/r \in R\}$.
(v) $A^C = \{(r, N_{A(r)}, I_{A(r)}, P_{A(r)})/r \in R\}$.

2.10 Proposition

Let $A, B, C \in PF(R)$.Then (i) If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$
(ii)$(A^C)^C = A$
(iii) Operations $\cap$ *and* $\cup$ are commutatiive, associative and distributive.
(iv) Operations $\cap$ *and* $\cup$ satisfy demorgons Laws.

3 Propertices of Picture Fuzzy Soft set

In this section,we study the concept of PFSS and define some relvant operation on a PFSS, namely subset,equal, complement,AND,OR and so on.Now we propose the definition of a PFSS and we give an illustrative example it.

3.1 Definition

Let R be an initial universe set and E a set of parameters. By a picture fuzzy soft set (PFSS) over R we mean a pair (δ, A) where $A \subseteq E$ and δ is a mapping given by $\delta : A \to PF(R)$.

3.2 Example

Consider a PFSS (δ, A) over R,where $R = \{h_1, h_2, h_3, h_4\}$ is the set of fours cases under consideration of a decision making to purchase, and $A = \{e_1, e_2, e_3\}$ is the set of pramaters where e_1 students for the parameter "Cheap" e_2 students for the parameter "Beautiful" e_3 students for the parameter "Good location"'the PFSS (δ, A) describes the "altractivencss of the house" to this decision maker.suppose that

R	$Cheap(e_1)$	$Beautiful(e_2)$	$Goodlocation(e_3)$
h_1	$(0.6, 0.3, 0.1)$	$(0.3, 0.5, 0.2)$	$(0.7, 0.5, 0.4)$
h_2	$(0.1, 0.4, 0.6)$	$(0.3, 0.4, 0.5)$	$(0.1, 0.7, 0.9)$
h_3	$(0.3, 0.7, 0.9)$	$(0.4, 0.6, 0.7)$	$(0.2, 0.5, 0.7)$
h_4	$(0.2, 0.5, 0.8)$	$(0.5, 0.7, 0.9)$	$(0.3, 0.5, 0.7)$

For convence of explanation ,we can also represent PFSS (δ, A) which is described in the above in matrix form as follows:

$$(\delta, A)= \begin{array}{c} \\ h_1 \\ h_2 \\ h_3 \\ h_4 \end{array} \begin{array}{c} \begin{array}{ccc} e_1 & e_2 & e_3 \end{array} \\ \begin{pmatrix} (0.6, 0.3, 0.1) & (0.3, 0.5, 0.2) & (0.7, 0.5, 0.4) \\ (0.1, 0.4, 0.6) & (0.3, 0.4, 0.5) & (0.1, 0.7, 0.9) \\ (0.3, 0.7, 0.9) & (0.4, 0.6, 0.7) & (0.2, 0.5, 0.7) \\ (0.2, 0.5, 0.8) & (0.5, 0.7, 0.9) & (0.3, 0.5, 0.7) \end{pmatrix} \end{array}$$

3.3 Definition

Let(δ, A) and (Δ, B) be two PFSS's over U. Then (δ, A) is called picture fuzzy soft subset of (Δ, B), denoted by $(\delta, A) \subseteq (\Delta, B)$ if

(i) $A \subseteq B$ and

(ii) $F(e) \subseteq G(e)$ for all $e \in A$.

3.4 Example

Let $U = \{C_1, C_2, C_3, C_4, C_5\}$ and $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$.Suppose (δ, A) and (Δ, B) are two PFSS's over U given by $A = \{e_1, e_2, e_3\}$ and $B = \{e_1, e_2, e_4, e_5\}$.

$$(\delta, A)= \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{array} \begin{array}{c} \begin{array}{ccc} e_1 & e_2 & e_3 \end{array} \\ \begin{pmatrix} (0.3, 0.2, 0.1) & (0.2, 0.7, 0.3) & (0.1, 0.5, 0.1) \\ (0.2, 0.2, 0.3) & (0.8, 0.6, 0.1) & (0.1, 0.6, 0.7) \\ (0.1, 0.3, 0.6) & (0.7, 0.3, 0.2) & (0.2, 0.3, 0.1) \\ (0.3, 0.1, 0.2) & (0.3, 0.1, 0.5) & (0.3, 0.7, 0.4) \\ (0.2, 0.1, 0.6) & (0.2, 0.3, 0.7) & (0.5, 0.6, 0.2) \end{pmatrix} \end{array}$$

$$
(\Delta, B)= \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{array} \begin{array}{c} \begin{array}{cccc} \quad e_1 \quad & \quad e_2 \quad & \quad e_4 \quad & \quad e_5 \quad \end{array} \\ \begin{pmatrix} (0.6,0.2,0.1) & (0.1,0.2,0.4) & (0.1,0.7,0.6) & (0.7,0.5,0.2) \\ (0.5,0.3,0.2) & (0.2,0.4,0.6) & (0.2,0.7,0.5) & (0.1,0.2,0.3) \\ (0.1,0.3,0.2) & (0.3,0.0,0.7) & (0.3,0.2,0.1) & (0.3,0.5,0.2) \\ (0.1,0.0,0.7) & (0.3,0.7,0.6) & (0.1,0.6,0.2) & (0.7,0.3,0.8) \\ (0.0,0.3,0.1) & (0.1,0.2,0.3) & (0.5,0.7,0.2) & (0.1,0.3,0.5) \end{pmatrix} \end{array}
$$

then δ_A is picture fuzzy soft subset of Δ_B.

3.5 Definition

Two PFS sets δ_A and Δ_B over U are called to be picture fuzzy softset equal,if and only if δ_A is a picture fuzzy soft subset of Δ_B and Δ_B is a picture fuzzy soft subset of δ_A.that is if $(\delta, A) \subseteq (\Delta, B)$ and $(\delta, B) \subseteq (\Delta, A)$ then $\delta_A = \Delta_B$.

3.6 Definition

Let δ_A be a PFS set over U.The complement of δ_A, denoted by δ_A^C, where $\delta_A^C : A \to PFS(U)$ is mapping given by $\delta_{(e)}^C = \{\delta_{(e)}\}^C$ for all $e \in A$.

3.7 Example

Consider the PFS set δ_A then the coplement of δ_A is represented as

$$
(\delta_A)^C= \begin{array}{c} \\ h_1 \\ h_2 \\ h_3 \\ h_4 \end{array} \begin{array}{c} \begin{array}{ccc} \quad e_1 \quad & \quad e_2 \quad & \quad e_3 \quad \end{array} \\ \begin{pmatrix} (0.1,0.3,0.6) & (0.2,0.5,0.3) & (0.4,0.5,0.7) \\ (0.6,0.4,0.9) & (0.5,0.4,0.3) & (0.9,0.7,0.1) \\ (0.9,0.7,0.3) & (0.7,0.6,0.4) & (0.7,0.5,0.2) \\ (0.8,0.5,0.2) & (0.9,0.7,0.5) & (0.7,0.5,0.3) \end{pmatrix} \end{array}
$$

By the suggestions given by Molodtsov,[14] we define the AND and OR operation on two PFS sets as follows.

3.8 Definition

Let δ_A and Δ_B are PFS sets over U. Then δ_A AND δ_B, denoted by $\delta_A \cap \delta_B$.

3.9 Example

Let $U = \{C_1, C_2, C_3, C_4\}$ and $E = \{e_1, e_2, e_3, e_4, e_5\}$.Take $A = \{e_1, e_2\}$ and $B = \{e_1, e_3, e_5\}$,define

$$\delta_A = \begin{array}{c c} & \begin{array}{cc} e_1 & e_2 \end{array} \\ \begin{array}{c} C_1 \\ C_2 \\ C_3 \\ C_4 \end{array} & \begin{pmatrix} (0.2, 0.3, 0.4) & (0.1, 0.3, 0.4) \\ (0.1, 0.4, 0.6) & (0.3, 0.2, 0.1) \\ (0.2, 0.0, 0.8) & (0.1, 0.3, 0.6) \\ (0.1, 0.2, 0.4) & (0.2, 0.4, 0.1) \end{pmatrix} \end{array} \text{ and}$$

$$\Delta_B = \begin{array}{c c} & \begin{array}{ccc} e_1 & e_3 & e_5 \end{array} \\ \begin{array}{c} h_1 \\ h_2 \\ h_3 \\ h_4 \end{array} & \begin{pmatrix} (0.1, 0.7, 0.6) & (0.2, 0.4, 0.7) & (0.1, 0.3, 0.6) \\ (0.2, 0.4, 0.3) & (0.1, 0.3, 0.8) & (0.2, 0.6, 0.7) \\ (0.1, 0.3, 0.6) & (0.4, 0.6, 0.2) & (0.1, 0.4, 0.6) \\ (0.7, 0.1, 0.8) & (0.7, 0.6, 0.3) & (0.1, 0.0, 0.2) \end{pmatrix} \end{array}$$

Here $\delta_A \cap \Delta_B = (H, C)$,where $C = A \cap B$ and $\forall e \in C$

$$H(e) = \begin{cases} \delta(e), & \text{if} \quad \text{x } e \in A - B \\ \Delta(e), & \text{if} \quad \text{x } e \in B - A \\ \delta(e) \cap \Delta(e), & \text{if} \quad e \in A \cap B \end{cases}$$

in this example

$$\delta_A \cap \Delta_B = (H, C) = \begin{array}{c c} & e_1 \\ \begin{array}{c} h_1 \\ h_2 \\ h_3 \\ h_4 \end{array} & \begin{pmatrix} (0.2, 0.3, 0.4) \\ (0.1, 0.4, 0.6) \\ (0.2, 0.0, 0.8) \\ (0.1, 0.2, 0.4) \end{pmatrix} \end{array}$$

3.10 Definition

Let δ_A and Δ_B are PFS sets over U. Then δ_A OR Δ_B, denoted by $\delta_A \cup \Delta_B$.

$\delta_A \cup \Delta_B =$

$$\begin{array}{c c} & \begin{array}{cccccc} (e_1, e_1) & (e_1, e_3) & (e_1, e_5) & (e_2, e_1) & (e_2, e_3) & (e_2, e_5) \end{array} \\ \begin{array}{c} C_1 \\ C_2 \\ C_3 \\ C_4 \end{array} & \begin{pmatrix} (0.2, 0.3, 0.4) & (0.2, 0.3, 0.7) & (0.1, 0.3, 0.6) & (0.1, 0.3, 0.6) & (0.1, 0.3, 0.7) & (0.1, 0.3, 0.6) \\ (0.1, 0.4, 0.6) & (0.1, 0.3, 0.8) & (0.1, 0.4, 0.7) & (0.2, 0.2, 0.3) & (0.1, 0.2, 0.8) & (0.2, 0.2, 0.7) \\ (0.1, 0.0, 0.8) & (0.2, 0.0, 0.8) & (0.1, 0.0, 0.8) & (0.1, 0.3, 0.6) & (0.1, 0.3, 0.6) & (0.1, 0.3, 0.6) \\ (0.1, 0.1, 0.8) & (0.1, 0.2, 0.4) & (0.1, 0.0, 0.4) & (0.2, 0.1, 0.8) & (0.2, 0.4, 0.3) & (0.1, 0.0, 0.2) \end{pmatrix} \end{array}$$

Here $\delta_A \cup \Delta_B = (H, C)$, where C=$(A \cup B)$ and for all $e \in C$.

$$K(e) = \begin{cases} \delta(e), & \text{if } \ \text{x } e \in A - B \\ \Delta(e), & \text{if } \ \text{x } e \in B - A \\ \delta(e) \cup \Delta(e), & \text{if } \ e \in A \cap B \end{cases}$$

$(\delta_A \cup \Delta_B) = (H, C)=$

$$\begin{array}{c c} & \begin{array}{cccc} e_1 & e_2 & e_3 & e_5 \end{array} \\ \begin{array}{c} C_1 \\ C_2 \\ C_3 \\ C_4 \end{array} & \begin{pmatrix} (0.2, 0.3, 0.4) & (0.1, 0.3, 0.4) & (0.2, 0.4, 0.7) & (0.1, 0.3, 0.6) \\ (0.1, 0.4, 0.6) & (0.4, 0.2, 0.1) & (0.1, 0.3, 0.8) & (0.2, 0.6, 0.7) \\ (0.0, 0.0, 0.8) & (0.1, 0.3, 0.6) & (0.4, 0.3, 0.2) & (0.1, 0.4, 0.6) \\ (0.1, 0.2, 0.4) & (0.2, 0.4, 0.1) & (0.7, 0.6, 0.3) & (0.1, 0.0, 0.2) \end{pmatrix} \end{array}$$

3.11 Theorem[Demorgon's Law]

Let δ_A and Δ_B be two PFS sets over U.Then (i) $(\delta_A \cap \Delta_B)^C = {\delta_A}^C \cup {\Delta_B}^C$ (ii) $(\delta_A \cup \Delta_B)^C = {\delta_A}^C \cap {\Delta_B}^C$.
Proof

$$\begin{aligned} (i)\text{Suppose that } (\delta, A) \cap (\Delta, B) &= (K, A \times B) \\ (\delta, A) \cap (\Delta, B)^C &= (K, A \times B)^C \\ &= (K^C, A \times B) \\ \text{Now}(\delta, A)^C \cup (\Delta, B)^C &= (\delta^C, A) \cup (\Delta^C, B) \\ &= (H, A \times B). \\ \text{Take } (\alpha, \beta) \in A \times B, \text{Therefore} & \\ K^C(\alpha, \beta) &= \{K(\alpha, \beta)\}^C \\ &= \{\delta(\alpha) \cap \Delta(\beta)\}^C \\ &= \delta^C(\alpha) \cup \Delta^C(\beta) \\ \text{agian } H(\alpha, \beta) &= \delta^C(\alpha) \cup \Delta^C(\beta) \\ H^C(\alpha, \beta) &= H(\alpha, \beta) \\ \text{The theorem is proved} & \end{aligned}$$

(ii) The result can be proved in a similar way.

3.12 Theorem

Union of two PFS sets δ_A and Δ_B is a PFS set.

Proof

We know that, Let (δ_A) and (Δ_B) are PFS sets over U. Then δ_A OR Δ_B, denoted by $\delta_A \cup \Delta_B$. and $\forall e \in C, e \in A \rightarrow B, or\ e \in B \rightarrow A$,then $K(e) = \delta(e)$ or K(e)=$\Delta(e)$. So, in either case,we have K(e) is a picture fuzzy soft set. If $e \in A \cap B$, for a fixed $x \in U$ without loss of generalety, suppose $\lambda_{\delta(e)}(x) \leq \lambda_{\Delta(e)}(x)$,

we have $\lambda_{K(e)}(x) + \mu_{K(e)}(x) + \gamma_{K(e)}(x)$=

$\max\{\lambda_{\delta(e)}(x), \lambda_{\Delta(e)}(x)\}$+$\min\{\mu_{\delta(e)}(x), \mu_{\Delta(e)}(x)\} + \min\{\gamma_{\delta(e)}(x), \gamma_{\Delta(e)}(x)\}$

=$\lambda_{\Delta(e)}(x) + \min\{\mu_{\delta(e)}(x), \mu_{\Delta(e)}(x)\} + \min\{\gamma_{\delta(e)}(x), \gamma_{\Delta(e)}(x)\}$

$\leq \lambda_{\Delta(e)}(x) + \mu_{\Delta(e)}(x) + \gamma_{\Delta(e)}(x) \leq 1$

There fore (K, C) is a PFS set.

Hence the proof.

3.13 Theorem

Intersection of two PFS sets δ_A and Δ_B is a PFS set.

Proof

we know that, Let δ_A and Δ_B are PFS sets over U. Thenδ_A OR Δ_B, denoted by $\delta_A \cap \Delta_B$. and $\forall e \in U$, without loss of generality supoose $\lambda_{\delta(e)}(x) \leq \lambda_{\Delta(e)}(x)$,

we have $\lambda_{K(e)}(x) + \mu_{K(e)}(x) + \gamma_{K(e)}(x)$=

Min$\{\lambda_{\delta(e)}(x); \lambda_{\Delta(e)}(x)\}$+Max$\{\mu_{\delta(e)}(x), \mu_{\Delta(e)}(x)\}$ + Max$\{\gamma_{\delta(e)}(x), \gamma_{\Delta(e)}(x)\}$

=$\lambda_{\Delta(e)}(x)$ + Min$\{\mu_{\delta(e)}(x), \mu_{\Delta(e)}(x)\}$ + Max$\{\gamma_{\delta(e)}(x), \gamma_{\Delta(e)}(x)\}$

$\leq \lambda_{\Delta(e)}(x) + \mu_{\Delta(e)}(x) + \gamma_{\Delta(e)}(x) \leq 1$

There fore (K, C) is a PFS set. Hence the proof.

3.14 Theorem

Let δ_A, Δ_B and K_C be PFS sets over U.Then (i) $\delta_A \cap \delta_B = \delta_A$ (ii) $\delta_A \cup \delta_A = \delta_A$. (iii) $(\delta_A \cap \Delta_B) = (\Delta_B \cup \delta_A)$ (iv) $(\delta_A \cup \Delta_B) = (\Delta_B \cap \delta_A)$.

(v) $(\delta_A \cup \Delta_B) \cup K_C = \delta \cup (\Delta_B \cup K_C)$ (vi) $(\delta_A \cap \Delta_B) \cap K_C = \delta \cap (\Delta_B \cap K_C)$.

Proof

The proofs are strightforward by using the definitions((3.5),(3.8),(3.10)) and Theorem(3.11)

(i)Let δ_A and Δ_B are PFS sets over U. Then δ_A AND δ_B, denoted by $\delta_A \cap \delta_B$.

(ii)Let δ_A and Δ_B are PFS sets over U. Thenδ_A OR δ_B, denoted by $\delta_A \cup \delta_B$

(iii) Let $A, B, C \in PF(R)$,then (i) If $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$

(ii)$(A^C)^C = A$

(iii) Operations $\cap$ and $\cup$ are commutatiive, associative and distributive.

3.15 Theorem[Distributive Law]

Let δ_A, Δ_B and K_C be PFS sets over U.Then (i) $\delta_A \cap (\Delta_B \cup K_C) = (\delta_A \cap \Delta_B) \cup (\delta_A \cap K_C)$ (ii)$\delta_A \cup (\Delta_B \cap K_C) = (\delta_A \cup \Delta_B) \cap (\delta_A \cup K_C)$

Proof

The proofs are strightforward by using the definition((3.5),(3.8),(3.10)) and Theorem(3.14).

(i)Let δ_A and Δ_B are PFS sets over U. Then δ_A AND δ_B, denoted by "$\delta_A \cap \delta_B$".

(ii)Let δ_A and Δ_B are PFS sets over U. Thenδ_A OR δ_B, denoted by "$\delta_A \cup \delta_B$"

(iii) Let $A, B, C \in PF(R)$,then (iv) If $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$

(v)$(A^C)^C = A$

(iii) Operations $\cap$ and $\cup$ are commutatiive, associative and distributive.

3.16 Theorem[Dual Law]

Let δ_A, Δ_B *and* K_C be PFS sets over U.Then (i) $(\delta_A \cap \Delta_B)^C = {\delta_A}^C \cup {\Delta_B}^C$, if and only if A=B

(ii) $(\delta_A \cup \Delta_B)^C = {\delta_A}^C \cap {\Delta_B}^C$. iff A=B

Proof

(i) If A=B, Then we have $\delta_A \cup \Delta_B = \delta_A \cup \Delta_A = (K, A)$. Now for all $e \in A$, $K(e) = \delta(e) \cup \Delta(e)$ Hence $(\delta_A \cap \Delta_B)^C = (\delta_A \cap \Delta_A)^C = (K, A)^C = (K^C, A)$ and $K^C(e) = (\delta(e) \cup \Delta(e))^C = \delta^C(e) \cap \Delta^C(e)$Ȧgain suppose that $(\delta_A \cap \Delta_B)^C = (\delta_A \cap \Delta_A)^C = (I, A)^C = (I^C, A) = (I, A)$ for all $e \in A$. $I(e) = \delta^C(e) \cup \Delta^C(e)$ we see that for all $e \in A.I(e) = K^C(e)$.therefore this result is true.

Conversely ,hypotheses $A \neq B$. Suppose that $\delta_A\ cup \delta_B$ =(K,C) where $C = A \cup B$ and for all $e \in C$.

$$K(e) = \begin{cases} \delta(e), & \text{if} \quad \text{x } e \in A - B \\ \Delta(e), & \text{if} \quad \text{x } e \in B - A \\ \delta(e) \cup \Delta(e), & \text{if} \quad e \in A \cap B \end{cases}$$

Thus $(\delta_A \cup \Delta_B)^C = K_C^C$ and

$$K^C(e) = \begin{cases} \delta^C(e), & \text{if} \quad \text{x } e \in A - B \\ \Delta^C(e), & \text{if} \quad \text{x } e \in B - A \\ \delta^C(e) \cap \Delta^C(e), & \text{if} \quad e \in A \cap B \end{cases}$$

Again suppose that $\delta_A^C \cap \Delta_B^C = (I, J)$.Where $J = A \cap B$ and $\forall e \in J$. $I(e) = \delta^C(e) \cap \Delta^C(e)$. obiviously, where $A \neq B$, we have $C = A \cup B \neq A \cap B = J$, so $K_C^C \neq I_J$. This is controdiction of over condition. $(\delta_A \cap \Delta_B)^C = {\delta_A}^C \cup {\Delta_B}^C$. Hence A=B.

(ii) This result can be proved in a similar way.

3.17 Remark

From the above theorem for dual theorem ,we know that Demorgon's laws are invalied for PFS sets with the different parameters sets, but they are true for PFS sets with the identical parameter set.

4 Picture Fuzzy Soft Relations and its Decision Making

In this section ,we construct picture fuzzy soft operator and a decision making method on relations.

Now we construct a decision making method on picture fuzzy soft relation by the following algorithm:

step-1 Input the picture fuzzy soft sets A and B

step-2 Obtain the picture fuzzy soft matrix R corresponding to contesection product of A and B respectively.

step-3 Compute the comparision table using the following formula $P_A(r) + I_A(r) - N_A(r)$.

step-4 Select the hight numerical grades from comparison table for each row.
step-5 Find the solve table which having the following form

R	(x_1, y_1)	...	...	(x_n, y_n)
$(Objects)$	h_1			
$(Highestgrade)$				

where X_n denotes the parameter of A and y_n denotes the parameter of B.
step-6 Compute the solve of each objects by taking the sum of these numerical grades.
step-7 Find m, for which $S_m = maxS_j$. then S_m is the hight score,if m has most then one values,you can choose any one value S_j.
Now we use this algorithm to find the best choice in decision making system.

4.1 Example

Let $U = \{u_1, u_2, u_3, u_4\}$ be the set of four cars.suppose that two friends wont to buy a car for a muutal friend among these four cars according to their choice parameters $E_1 = \{x_1, x_2, x_3\}$={Expensive, moderate, inexpensive} and $E_2 = \{y_1, y_2, y_3\}$={Green,Black,Red} respectively, then we select a car on the basis of the sets of friends parameters by using the picture fuzzy soft relation decision making method.
step-1 We input the picture fuzzy soft set A and B as

$$A= \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cccc} \quad u_1 \quad & \quad u_2 \quad & \quad u_3 \quad & \quad u_4 \quad \end{array} \\ \begin{pmatrix} (0.3,0.6,0.4) & (0.2,0.4,0.5) & (0.3,0.6,0.7) & (0.3,0.5,0.7) \\ (0.1,0.3,0.4) & (0.1,0.3,0.3) & (0.2,0.4,0.5) & (0.1,0.3,0.9) \\ (0.3,0.3,0.4) & (0.4,0.6,0.7) & (0.3,0.4,0.5) & (0.3,0.7,0.9) \end{pmatrix} \end{array}$$

$$\text{and } B= \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cccc} \quad u_1 \quad & \quad u_2 \quad & \quad u_3 \quad & \quad u_4 \quad \end{array} \\ \begin{pmatrix} (0.4,0.6,0.7) & (0.2,0.4,0.5) & (0.4,0.4,0.6) & (0.3,0.5,0.9) \\ (0.2,0.3,0.5) & (0.3,0.5,0.6) & (0.2,0.4,0.5) & (0.1,0.5,0.9) \\ (0.2,0.3,0.4) & (0.2,0.8,0.8) & (0.3,0.6,0.6) & (0.1,0.4,0.7) \end{pmatrix} \end{array}$$

step-2 we obtain the picture fuzzy soft matrix R corresponding to cartesion product of A and B respectively

R	u_1	u_2	u_3	u_4
(x_1, y_1)	$(0.3, 0.6, 0.7)$	$(0.2, 0.4, 0.5)$	$(0.3, 0.5, 0.7)$	$(0.3, 0.5, 0.7)$
(x_1, y_2)	$(0.1, 0.45, 0.4)$	$(0.2, 0.45, 0.6)$	$(0.2, 0.5, 0.7)$	$(0.1, 0.45, 0.9)$
(x_1, y_3)	$(0.2, 0.45, 0.7)$	$(0.2, 0.6, 0.8)$	$(0.3, 0.6, 0.7)$	$(0.1, 0.45, 0.7)$
(x_2, y_1)	$(0.1, 0.45, 0.7)$	$(0.1, 0.4, 0.6)$	$(0.2, 0.4, 0.6)$	$(0.1, 0.4, 0.9)$
(x_2, y_2)	$(0.1, 0.3, 0.5)$	$(0.1, 0.4, 0.6)$	$(0.2, 0.4, 0.5)$	$(0.1, 0.4, 0.9)$
(x_2, y_3)	$(0.1, 0.3, 0.4)$	$(0.1, 0.055, 0.8)$	$(0.2, 0.5, 0.6)$	$(0.1, 0.35, 0.9)$
(x_3, y_1)	$(0.3, 0.4, 0.7)$	$(0.2, 0.5, 0.7)$	$(0.3, 0.4, 0.6)$	$(0.3, 0.06, 0.9)$
(x_3, y_2)	$(0.2, 0.3, 0.5)$	$(0.3, 0.055, 0.7)$	$(0.2, 0.4, 0.5)$	$(0.3, 0.6, 0.7)$
(x_3, y_3)	$(0.2, 0.3, 0.4)$	$(0.2, 0.7, 0.8)$	$(0.3, 0.5, 0.6)$	$(0.3, 0.6, 0.9)$

Table -2 picture fuzzy soft relational matrix R.

Step-3 By using table-1, we compute the comparison table as

R	u_1	u_2	u_3	u_4
(x_1, y_1)	0.2	0.1	0.1	0.1
(x_1, y_2)	-0.15	0.05	0	-0.35
(x_1, y_3)	-0.05	0.00	0.2	-0.15
(x_2, y_1)	-0.15	-0.1	0.00	-0.4
(x_2, y_2)	-0.1	-0.1	0.1	-0.4
(x_2, y_3)	0.00	-0.645	0.1	-0.45
(x_3, y_1)	0.05	0.00	0.1	-0.54
(x_3, y_2)	0.00	-0.345	0.1	0.2
(x_3, y_3)	0.1	0.1	0.2	0.2

Table -2 comparsion table (P+I-N).

step-4 we select the highest numerical numerical grade from step-3 for each row

R	u_1	u_2	u_3	u_4
(x_1, y_1)	0.2	0.1	0.1	0.1
(x_1, y_2)	-0.15	0.05	0	-0.35
(x_1, y_3)	-0.05	0.00	0.2	-0.15
(x_2, y_1)	-0.15	-0.1	0.00	-0.4
(x_2, y_2)	-0.1	-0.1	0.1	-0.4
(x_2, y_3)	0.00	-0.645	0.1	-0.45
(x_3, y_1)	0.05	0.00	0.1	-0.54
(x_3, y_2)	0.00	-0.345	0.1	0.2
(x_3, y_3)	0.1	0.1	0.2	-0.745

.

Table-3 Highest value of each row

step-5 we find the score table which have the following form

R	u_1	u_2	u_3	u_4
(x_1, y_1)	0.2	$-$	$-$	$-$
(x_1, y_2)	$-$	0.05	$-$	$-$
(x_1, y_3)	$-$	$-$	0.2	$-$
(x_2, y_1)	$-$	$-$	0	$-$
(x_2, y_2)	$-$	$-$	0.1	$-$
(x_2, y_3)	$-$	$-$	0.1	$-$
(x_3, y_1)	$-$	$-$	0.1	$-$
(x_3, y_2)	$-$	$-$	$-$	0.2
(x_3, y_3)	$-$	$-$	0.2	$-$

.

Table -4 is score table.

step-6 we compute the score of each objects by taking the form of numerical grades as;

$u_1 = 0.2$,

$u_2 = 0.05$,

$u_3 = 0.2 + 0 + 0.1 + 0.1 + 0.2 = 0.7$,

$u_4 = 0.2$

step-7 The maximum value of the score value is $S_j = 0.7$,so the two friends will select the car with the highest score ,hence they will choose car u_3. with parameter either expensive carwith red or expensive with red.

5 Conclusion

We have defined a picture fuzzy soft set with some special operations and proved various result based on the picture fuzzy soft set.Finaly we study the decision making approach for solving picture fuzzy soft matrix under relational concepts.one can obtian the similar result in fermatean fuzzy soft set and pythogoner fuzzy soft sets.

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