Economic Model Using Double Sampling Plan $(n_2 = 2n_1)$ Based On Binomial Distribution

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Abstract: *An Acceptance sampling plan is the overall design for accepting or rejecting a lot based on samples. Acceptance sampling plan plays a vital role in quality control. In this paper, we design an economic model to determine the double sampling plan ($n_2 = 2n_1$) that minimizes the consumer's and the producer's total quality cost while satisfying both the producer's and consumer's risk and quality requirements. Examples are show that the optimal sampling plan is quite sensitive to the producer's product quality.*

Keywords: Acceptance Sampling Plan, Double Sampling Plan , Economic Design, Producer Risk, Consumer Risk

1. INTRODUCTION

Acceptance sampling is a necessary part of manufacturing and may be applied to incoming materials to partially finished product at various intermediate stages of the manufacturing process, and to final product. Acceptance inspection may also be carried out by the purchases of manufactured products. Much of this acceptance inspection is by sampling. Sampling inspection has a number of psychological advantages over 100% inspections. Good sampling acceptance procedure may often contribute to this objective pressure for quality improvement than can be exerted with 100% inspection. Some sampling schemes also provide a better basis for diagnosis of quality troubles than is common with 100% inspections. It should be recognized that although modern sampling acceptance procedures are generally superior to traditional sampling methods.

A.Kanagaw Okayama, Japan H. Ohta [1] provides determination of Sample Sizes for Double Sampling Attribute Plans. A table for the construction and evaluation of matched sets of single, double and multiple sampling plans developed by Shilling and Johnson [2]. There are number of tables available to design a double sampling plan including Dodge and Romig [3] which provide Double sampling plans with minimum Average Total inspection. Further, Soundararajan and Arumainayagam [4] have provided tables for easy selection of double sampling plan indexed by AQL, Average outgoing Quality Level (AOQL) and Limiting Quality Level (LQL). Ferrell, W.G., and Chhoker A [5] developed the design of economically optimal acceptance sampling plans with inspection error and Lie –Fern Hsu, and Jia-Tzer Hsu (2012) provided Economic Design of Acceptance Sampling Plans in a Two-Stage Supply Chain. Mohammad Saber Fallah Nezhad, Ahmad Ahmadi Yazdi, Parvin Abdollahi, and Muhammad Aslam [6] developed the Design of Economic Optimal Double Sampling Design with Zero Acceptance Numbers.

2. Authors Contribution

Wetherill and Chiu [7] reviewed few major principles of acceptance sampling plan with emphasis on the economic aspects. A plot of the ASN, versus the incoming defect level p, describes the sampling efficiency of a given sampling plan. Their research indicated that the major approaches for designing an economic acceptance sampling plan include Bayesian approach , Minimax approach and Semi-economic approach. An economic model to assist in the selection of minimum cost acceptance sampling plans by variables. The quadratic developed by Tagaras [8]. Jia-Tzer Hsu and Lie-Fern Hsu [9] developed an+ economic design of single sample acceptance sampling plans has integrated the producer's and the consumer's risk requirements into the design of the model. Ksenija Dumičić and Berislav Žmuk [10] which provide Decision making based on single and double acceptance sampling plans for assessing quality of lots.

This paper is to design an economic model to determine the Double sampling plan with ($n_2 = 2n_1$) that minimizes the producer’s total cost while satisfying both the consumer’s and the producer’s risk and quality requirements. In this paper is organized as follows, formulates the optimization problem for the economic design with ($n_2 = 2n_1$), numerical examples to explain the effects of quality and costs on the double sampling plan.

3. EVALUATION AND DESIGN OF DOUBLE SAMPLING PLAN WITH ($n_2 = 2n_1$)

3.1 Double sampling plan

Double sampling plan was given by Dodge and Romig (1941), is based on two samples. If the first sample does not satisfy the acceptance criteria, then the second sample is taken and the decision is provided by both the samples combined. If the size of the first sample taken in double sampling plan is less than that of the sample size taken in a single sampling plan for similar lot size and the decision is made on the first sample size alone, then it is more beneficial than the single sampling plan. Further in double sampling plan, the ASN and ATI values also explains that smaller sample size is sufficient to be inspected in average than that of a single sampling plan.

A double sampling with n_1 *and* n_2 being the size of the first and second sample respectively, c_1 *and* c_2 be the acceptance numbers of the first and second sample and d_1 be the number of non-conformities in the first sample and $D = d_1 + d_2$ be the number of non-conformities in the first and second sample

3.2 Operating Procedure

- From a lot , take a random sample of size n_1 and count the number of nonconforming units d_1
- If $d_1 \le c_1$ accept the lot
- If $d_1 > c_1$, reject the lot

- If $c_1 + 1 < d_1 \leq c_2$, take a second random sample of size n_2 and count the number of nonconforming units D
- If $D \leq c_2$, accept the lot
- If $D > c_2$, reject the lot

3.3 Performance measurements

The double sampling plan has the following performance measurements:

$$P_a(p) = \sum_{i=0}^{c_1} p_i + \sum_{j=c_1+1}^{c_2} p_j \sum_{i=0}^{c_2-j} q_i$$

$$\text{where } p_i = \frac{{}^{n_1}c_x\, p^x q^{n_1-x}}{1-q^{n_1}}, \quad q_i = \frac{{}^{n_2}c_x\, p^x q^{n_2-x}}{1-q^{n_2}}, \quad \because n_2 = 2n_1$$

$$ATI = n_1 p_{a_1} + (n_1 + n_2) p_{a2} + N(1 - p_a)$$

$$ASN = n_1 + P(\text{taking } n_1) n_2$$

$$AOQ = \frac{P_a p'(N-n)}{N}$$

Let Dn denotes the defective items not detected and Dd denote the defective items detected

$$D_n = P_a(p)(N-n)p$$

$$D_d = np + (1 - P_a(p))(N-n)p$$

To derive the total quality cost per lot for a given sampling plan, we define the following cost Parameters:

Ci: Inspection cost per item

Cf : Internal failure cost;

Co: The cost of an outgoing defective unit, i.e. the post-sale failure cost.

The economic sampling plan can be found through the following mathematical model:

$$\text{Minimize } TC = C_i\ ATI + C_f\ D_d + C_0\ D_n$$

$$\text{Subject to condition}$$

$$1 - P_a(AQL) \leq \alpha$$

$$P_a(LTPD) \leq \beta$$

4. NUMERICAL EXAMPLES AND DISCUSSION

Table 1 shows the sensitivity analysis of the Double sampling plan with $n_2 = 2n_1$. The AOQ value decreases when p increases, different levels of 'f' and 'i' values. When the p-value is more than 0.07 the AOQ value is near to zero, so 100% inspection needed.

Table 1: Sensitivity analysis of the Double sampling plan with $n_2 = 2n_1$

P	TC	n1	n2	AOQ	ATI	Dd	Dn	Pa
0.01	936.3	150	300	0.1293	905.9990	8.7075	1.2925	0.2350
0.02	1006.0	152	304	0.0601	956.2447	18.7856	1.2144	0.1116
0.03	1048.9	154	308	0.0231	983.2052	29.2941	0.7059	0.0437
0.04	1077.1	156	312	0.0073	994.6723	39.6985	0.3015	0.0142
0.05	1099.4	158	316	0.0020	998.5532	49.8966	0.1034	0.0039
0.06	1119.9	160	320	0.0005	999.6542	59.9700	0.0300	0.0010
0.07	1140.0	162	324	0.0001	999.9260	69.9924	0.0076	0.0002
0.08	1160.0	164	328	0.0000	999.9856	79.9983	0.0017	0.0000
0.09	1180.0	166	332	0.0000	999.9975	89.9997	0.0003	0.0000
0.1	1200.0	168	336	0.0000	999.9996	99.9999	0.0001	0.0000
0.11	1220.0	170	340	0.0000	999.9999	110.0000	0.0000	0.0000
0.12	1240.0	172	344	0.0000	1000.0000	120.0000	0.0000	0.0000
0.13	1260.0	174	348	0.0000	1000.0000	130.0000	0.0000	0.0000
0.14	1280.0	176	352	0.0000	1000.0000	140.0000	0.0000	0.0000
0.15	1300.0	178	356	0.0000	1000.0000	150.0000	0.0000	0.0000
0.16	1320.0	180	360	0.0000	1000.0000	160.0000	0.0000	0.0000

As shown in Table 2, one can see the internal failure cost Cf is relatively insensitive to the optimal sampling plan. When Cf is increasing then the Total production cost is also increase based on Cf, Dn, Dd, and ATI values.

Table 2: Internal failure cost Cf is relatively insensitive to the optimal sampling plan

n1	n2	ATI	C0	Cf	Dd	Dn	TC
150	300	905.999	10	0.1	8.7075	1.2925	919.8
152	304	956.245	10	0.2	18.7856	1.2144	972.1
154	308	983.205	10	0.3	29.2941	0.7059	999.1
156	312	994.672	10	0.4	39.6985	0.3015	1013.6
158	316	998.553	10	0.5	49.8966	0.1034	1024.5
160	320	999.654	10	0.6	59.9700	0.0300	1035.9
162	324	999.926	10	0.7	69.9924	0.0076	1049.0
164	328	999.986	10	0.8	79.9983	0.0017	1064.0
166	332	999.997	10	0.9	89.9997	0.0003	1081.0
168	336	1000	10	1	99.9999	0.0001	1100.0
170	340	1000	10	1.1	110.0000	0.0000	1121.0
172	344	1000	10	1.2	120.0000	0.0000	1144.0
174	348	1000	10	1.3	130.0000	0.0000	1169.0
176	352	1000	10	1.4	140.0000	0.0000	1196.0
178	356	1000	10	1.5	150.0000	0.0000	1225.0
180	360	1000	10	1.6	160.0000	0.0000	1256.0

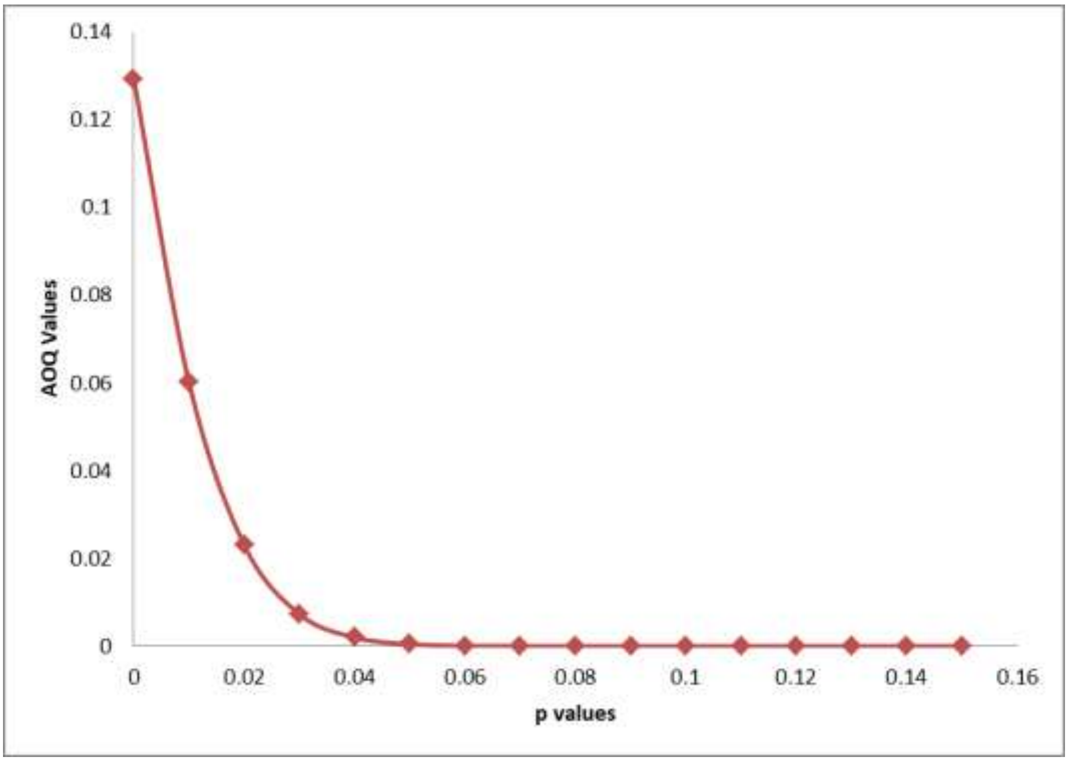
Table 3 shows the sensitivity analysis of the inspection cost Ci. If Ci ≤ 0.2, compared to the failure cost (Cf) the inspection cost (Ci) is very low.

Table 3: Sensitivity analysis of the inspection cost Ci

Ci	**TC**	**n1**	**n2**	**C0**	**AOQ**	**ATI**	**Dd**	**Dn**
0.1	190.6	150	300	10	0.1293	906.00	8.7075	1.2925
0.2	391.2	152	304	10	0.0601	956.24	18.7856	1.2144
0.3	595.0	154	308	10	0.0231	983.21	29.2941	0.7059
0.4	797.9	156	312	10	0.0073	994.67	39.6985	0.3015
0.5	999.3	158	316	10	0.0020	998.55	49.8966	0.1034
0.6	1199.8	160	320	10	0.0005	999.65	59.9700	0.0300
0.7	1399.9	162	324	10	0.0001	999.93	69.9924	0.0076
0.8	1600.0	164	328	10	0.0000	999.99	79.9983	0.0017
0.9	1800.0	166	332	10	0.0000	1000.00	89.9997	0.0003
1	2000.0	168	336	10	0.0000	1000.00	99.9999	0.0001
1.1	2200.0	170	340	10	0.0000	1000.00	110.0000	0.0000
1.2	2400.0	172	344	10	0.0000	1000.00	120.0000	0.0000
1.3	2600.0	174	348	10	0.0000	1000.00	130.0000	0.0000
1.4	2800.0	176	352	10	0.0000	1000.00	140.0000	0.0000
1.5	3000.0	178	356	10	0.0000	1000.00	150.0000	0.0000
1.6	3200.0	180	360	10	0.0000	1000.00	160.0000	0.0000

Fig.1 shows the AOQ curve is similar to the OC curve. AOQ value decreases when p-value increases. In certain point, the AOQ value is reduced to zero and then 100% inspection of the entire lot is needed.

Fig.1: AOQ curve



5. CONCLUSION

In this paper, we design an economic model to determine the double sampling plan ($n_2 = 2n_1$) that minimizes the consumer's and the producer's total quality cost while satisfying both the producer's and consumer's risk and quality requirements. The numerical illustrations concluded that double sampling plan with $n_2 = 2n_1$ have better performance in arriving measures. AOQ curve was constructed which shows the efficiency of the double sampling plan with $n_2 = 2n_1$.

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