Existence of solutions for fixed point theorem
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1. Introduction

The development of a mathematical model based on diffusion has received a great dealof attention in recent years, many scientist and mathematician have tried to apply basicknowledge about the differential equation and the boundary condition to explain anapproximate the diffusion and reaction model.The subject of fractional calculus attracted much attentions and is rapidly growing area of research because of itsnumerous applications in engineering and scientific disciplines such as signal processing, nonlinear control theory, viscoelasticity, optimization theory [1], controlled thermonuclear fusion, chemistry, nonlinear biological systems, mechanics,electric networks, fluid dynamics, diffusion, oscillation, relaxation, turbulence, stochastic dynamical system, plasmaphysics, polymer physics, chemical physics, astrophysics, and economics. Therefore, it deserves an independent theoryparallel to the theory of ordinary differential equations (DEs).

In the development of non-linear analysis, fixed point theory plays an important role. Also, it has been widely used in different branches of engineering and sciences. Banach fixed point theory is a essential part of mathematical analysis because of its applications in various area such as variational and linear inequalities, improvement and approximation theory. The fixed-point theorem in diffusion equations plays a significant role to construct methods to solve the problems in sciences and mathematics. Although Banach fixed point theory is a vast field of study and is capable of solving diffusion equations. The main motive of the research is solving the diffusion equations by Banach fixed point theorems and Adomian decomposition method. To analysis the drawbacks of the other fixed-point theorems and different solving methods, the related works are reviewed in this paper.

2. Review related on fixed point theorems

The fixed-point theorems are utilized to solve the diffusion equation problems. Many researchers are developed fixed-point theorems to solve the diffusion equations. Some of the problems are reviewed in this section.

2.1. Brouwer's Fixed Point Theorem

In this section, the related works of the Brouwer,s fixed point theorems are reviewed. Many works are available to analysis the Brouwer,s fixed point theorems, some of the works are reviewed.

Vasco Brattka*et al.,* [2] have presented the computational content of the Brouwer Fixed PointTheorem in the Weihrauch lattice. Connected choice was the operation that findsa point in a non-empty connected closed set given by negative information.One of main results were that for any fixed dimension the Brouwer FixedPoint Theorem of that dimension was computably equivalent to connected choiceof the Euclidean unit cube of the same dimension. Another main result was thatconnected choice was complete for dimension greater than or equal to two in thesense that it was computably equivalent to Weak K″onig's Lemma. While canpresent two independent proofs for dimension three and upwards that were eitherbased on a simple geometric construction or a combinatorial argument, theproof for dimension two was based on a more involved inverse limit construction.The connected choice operation in dimension one was known to be equivalent tothe Intermediate Value Theorem; prove that this problem was not idempotentin contrast to the case of dimension two and upwards. Here, also prove thatLipschitz continuity with Lipschitz constants strictly larger than one does notsimplify finding fixed points. Finally, prove that finding a connectednesscomponent of a closed subset of the Euclidean unit cube of any dimensiongreater or equal to one was equivalent to Weak K″onig's Lemma. Jean Mawhin*et al.,* [3] have presented Simple Proofs of the Hadamard and Poincare–Miranda Theorems Using the brouwer fixed point theorem. The theorems of Hadamard and of Poincare–Miranda give sufficient conditions for the existence of at least one zero for some continuous mappings from a Euclidean space intoitself. The note gives elementary and brief derivations of these theorems from the

Brouwerfixed point theorem. Brouwer's original proof was topological and quite sophisticated, based on somefixed-point theorems on spheres proved with the help of the topological degree introduced in the same paper. Yasuhito Tanaka *et al.,* [4] have presented two individuals and three alternatives the Arrowimpossibility theorem which equivalent to the Brouwer fixed point theorem on a 2- dimensional ball (circle) using elementary concepts and techniques of algebraictopology, in particular, homology groups of simplicial complexes, homomorphisms of homology groups. This approach may be applied to other social choice problems such asWilsons impossibility theorem, the GibbardSatterthwaite theoremand Amartya Sens liberalparadox. LudwikDąbrowski*et al.,* [5] have presented some results and conjectures on a generalization to the noncommutative setupof the Brouwer fixed-point theorem from the BorsukUlam theorem perspective. The BorsukUlam theorem, a fundamental theorem of topology, was often formulated in one of three equivalentversions, that regard continuous maps whose either domain, or codomain, were spheres.

James F peters *et al.,* [6] have presented shape boundary regions in descriptive proximity forms of CW (Closure-finite Weak) spaces as a source of amiable fixedsubsets as well as almost amiable fixed subsets of descriptive proximally continuous (dpc) maps. A dpc map was an extension of an Efremovičc-Smirnovproximally continuous (pc) map introduced during the early-1950s by V.A.Efremovic and Yu. M. Smirnov. Amiable fixed sets and the Betti numbers oftheir free Abelian group representations wer derived from dpc's relative to thedescription of the boundary region of the sets. Almost amiable fixed sets were derived from dpc's by relaxing the matching description requirement for thedescriptive closeness of the sets. This relaxed form of amiable fixed sets workswell for applications in which closeness of fixed sets was approximate rather thanexact. A number of examples of amiable fixed sets were given in terms of wideribbons.

2.2. Schauder's Fixed Point Theorem

Tiexinguo*et al.,* [7] have presented a noncompact generalization of the classical Schauder fixed point theorem for the development and financial applications of RN modules. Motivated by the randomized version of the classicalBolzano–Weierstrass

theorem, first introduce the two notions of a random sequentially compact set and a random sequentially continuous mappingunder the (ε, λ)–topology and further establish their corresponding characterizations under the locally L0–convex topology so that can treat the fixed-point problems under the two kinds of topologies in a unified way. Then prove desired Schauder fixed point theorem that in a σ–stable RN moduleevery continuous (under either topology) σ–stable mapping T from a randomsequentially compact closed L0–convex subset G to G has a fixed point. Thewhole idea to prove the fixedpoint theorem was to find an approximate fixedpoint of T, but, since G was not compact in general, realizing such an idea inthe random setting forces us to construct the corresponding Schauder projection in a subtle way and carry out countably many decompositions for T sothat can first obtain an approximate fixed point for each decompositionand eventually one for T by the countable concatenation skill.

Thenmozhi Shanmugam *et al.,* [8] have presented the existence of solutions for the particular type of the eighth-orderboundary value problem. Here, prove results using classical version of Leray–Schauder nonlinearalternative fixed-point theorem. Also produce a few examples to illustrate results. The obtain the results to prove the existence of positive solution for the eighth-orderboundary value problem with the help of the classical version of Leray–Schauder alternative fixed-pointtheorem. By applying these results, one can easily verify that whether the given boundary value problem was solvable or not. S. Chatterjee *et al.,* [9] have presented the Schauder-type fixed point theorem for the class of fuzzycontinuous, as well as fuzzy compact operators. This established in a fuzzy normed linear space (fnls)whose underlying t-norm was left-continuous at (1, 1). In the fuzzy setting, the concept of the measureof non-compactness was introduced, and some basic properties of the measure of non-compactness were investigated. Darbo's generalization of the Schauder-type fixed point theorem was developedfor the class of ψ −set contractions. This theorem was proven by using the idea of the measureof non-compactness. Ehsan Pourhadi*et al.,* [10] have presented Combining the Wavelet Basis with the Schauder Fixed PointTheorem to solving Nonlinear p-Adic Pseudo-differential Equations. These mathematical studies were motivated byapplications to problems of geophysics (fluids flow through capillary networks inporous disordered media) and the

turbulence theory. In this article, using this wavelettechnique in combination with the Schauder fixed point theorem, study the solvability of nonlinear equations with mixed derivatives, p-adic (fractional) spatial andreal time derivatives. Furthermore, in the linear case find the exact solution for theCauchy problem. Some examples were provided to illustrate the main results. Maryam Ramezani*et al.,* [11] have presented a new Version of Schauder and Petryshyn Type FixedPoint Theorems in S-Modular Function Spaces. In this article, using the conditions of Taleb-Hanebaly's theorem in a modular space wherethe modular was s-convex and symmetric with respect to the ordinate axis, prove a new generalizedmodular version of the Schauder and Petryshyn fixed point theorems for nonexpansive mappings ins-convex sets.

2.3. Schauder-Tychonoff

Erika hausenblas*et al.,* [12] have presented the existence and pathwise uniquenessof a nonnegative martingale solution to the stochastic evolution system of nonlinearadvection-diffusion equations proposed by Klausmeier with Gaussian multiplicativenoise. On the other hand, present and verify a general stochastic version of theSchauder-Tychonoff fixed point theorem, as its application was an essential step forshowing existence of the solution to the stochastic Klausmeier system. The analysisof the system was based both on variational and semigroup techniques. Here, also discussadditional regularity properties of the solution.

Mariusz Gil *et al.,* [13] have presented Schauder-Tychonoff Fixed-Point Theorem inTheory of Superconductivity. This paper was presented the existence of mild solutions to the time-dependent Ginzburg-Landau ((TDGL), for short) equations on an unboundedinterval. The rapidity of the growth of those solutions was characterized. Here, investigate the local and global attractivity of solutionsof TDGL equations and describe their asymptotic behaviour. The TDGL equations model the state of a superconducting samplein a magnetic field near critical temperature. This paper was based on the theory of Banach space, Frechet space, and Sobolew space.

Toufic El Arwadi*et al.,* [14] have presented some fixed-point theorems of theSchauder and Krasnoselskii type in a Frechet topological vector spaceE. Here,proves a

fixed-point theorem which was for every weakly compactmap from a closed bounded convex subset of a Frechet topological vectorspace having the Dunford–Pettis property into itself has a fixed point. Yasuhito Tanaka *et al.,* [15] have developed a constructiveversion of Tychonosoxed point theorem for a locally convex space using a constructive version of KKM (Knaster, Kuratowski and Mazurkiewicz) lemma, anda constructive version of Schaudersoxed point theorem for a Banach space as acorollary to that of Tychonos theorem.It was often demonstrated that Brouwers oxed point theorem cannot be constructively or computably proved. Therefore, Tychono§ís and Schaudersoxedpoint theorems also cannot be constructively proved. On the other hand, however, Sperners lemma which was used to prove Brouwers theorem can be constructively proved. Some authors have presented a constructive (or an approximate)version of Brouwers theorem using Sperners lemma.

A Khastan*et al.,* [16] have presented Schauder fixed-point theorem in semi-linearspaces and its application to fractionaldifferential equations with uncertainty. This study was the existence of solution for nonlinear fuzzy differential equations offractional order involving the Riemann-Liouville derivative. The Schauder Fixed-Point Theorem was one of the most celebrated results in Fixed-PointTheory and it states that any compact convex nonempty subset of a normed space has thefixed-point property. It was also valid in locally convex spaces. Recently, this Schauder fixed-pointtheorem has been generalized to semi-linear spaces.

2.4. Banach's fixed point theorem

ErdalKarapınar*et al.,* [17] have presented new fixed-point theorem which was inspired from both Caristi and Banach. In fixed point theory, the approaches of the renowned results of Caristi and Banach were quitedifferent and the structures of the corresponding proofs varies. In this short note, presented a newfixed-point theorem that was inspired from these two famous results. Here, aim to present results in the largest framework, b-metric space, instead of standard metricspace. The concept of b-metric has been discovered several times by different authors with distinctnames, such as quasi-metric, generalized metric and so on. On the other hand, this concept becamepopular after

the interesting papers of Bakhtin and Czerwik. For more details in b-metric spaceand advances in fixed point theory in the setting of b-metric spaces. janusz*et al.,* [18] have presented a fixed-point theorem in 2-Banach spaces andshow its applications to the Ulam stability of functional equations. The obtained stability results concern both some single variable equations and the most important functional equationin several variables, namely, the Cauchy equation. Moreover, a few corollaries correspondingto some known hyperstability outcomes were presented. JanuszBrzdek*et al.,* [19] have presented a fixed-point theorem foroperators acting on some classes of functions, with values in n-Banach spaces. Here, also present applications of it to Ulam stability of eigenvectors and somefunctional and difference equations.

Hao Liu *et al.,* [20] have presented the concept of quasicontractions on cone metric spaces with Banach algebras, and by a new method of proof,prove the existence and uniqueness of fixed points of such mappings. The main result generalizes the well-known theorem of Ciric. ArefJeribi*et al.,* [21] have presented one fixed point theorems of a 2×2 blockoperator matrix defined on nonempty bounded closed convex subsets of Banachalgebras, where the entries were nonlinear operators. Furthermore, apply theobtained results to a coupled system of nonlinear equations. Fixed point theorems were powerful techniques in finite-dimensional vector space, however, if can obtain theorems for operator matrices with nonlinear block acting oninfinite-dimensional vector spaces and particularly on Banach algebras gain much morepowerful once, can model integro-differential equations in such a way that to find afixed point of continuous map defined on functional spaces, it was equivalent to solve thissystem of equations that in general was quite hard to prove the existence of solutions usingstandard arguments.

Nabil Mlaiki*et al.,* [22] have developed fixed-point theorems using the set of simulation functionson an S-metric space with some illustrative examples. The results were stronger than some knownfixed-point results. Furthermore, give an application to the fixed-circle problem with respect to asimulation function. Showing the existence and uniqueness of a fixed point has many applications, in different fields,such as computer sciences, engineering, etc.Most of the work after was basically a generalization of the work of Banach.These generalizations include more general metric spaces, or more

general contractions, etc,which were important due to the fact that the more general the metric space, the larger the class,which implies that the obtained results can be applied in more different fields to solve unsolvedproblems. Moreover, these generalizations do not just include metric spaces; they include contractions,as well.

Naimatullah*et al.,* [23] have presented Fixed Point Theorems in Complex Valued Extended b-MetricSpaces. In this article, inspired by the concepts of extended b-metric spaces, we introduce the notionof complex valued extended b-metric spaces. Using this new idea, some fixed-point theorems involvingrational contractive inequalities were proved. The established results herein augment several significant worksin the comparable literature. Fixed point theory was a well-known and vast field of research in mathematical sciences. This field was known asthe mixture of analysis which includes topology, geometry and algebra. In particular, fixed point technique was acentral tool in the study of non-linear analysis. In this area, a huge involvement has been made by Banach,who gave the notion of contraction mapping due to a complete metric space to locate fixed point of the specifiedfunction. The classical fixed-point theorem due to Banach has been generalized by many researchers invarious ways and the references therein.

2.5. Arzela Ascoli theorem

Ronglu Li *et al.,* [24] have developed classical Arzela–Ascoli theorem and its typical modern formulation, haveimproved the sufficiency part by weakening the compactness of the domain space, andthe necessity part was improved by strengthening the necessity part of the classical version.The classical Arzela-Ascoli theorem plays an important role infunctional analysis.During the work on the free group case realizedthat for discrete groups, whichhave the property, named rapid decay, it was quite easy to construct metrics on thestate space of the reduced C2-algebra such that these metrics have all the propertiesaskedfor.

Mohammed Bachir*et al.,* [25] have presented metrication of probabilistic spaces. More precisely, given such a space and provided that the triangle function was continuous, exhibit an explicit and canonical metric on G such that the associated

topology was homeomorphic to the so-called strong topology. As applications, make advantage of this explicit metric to present some fixed-point theorems on such probabilistic metric structures and prove a probabilistic version of the Arzela-Ascoli theorem.

Christina sormani*et al.,* [26] have presented two Arzela-Ascoli Theorems proven for intrinsic flatconverging sequences of manifolds: one for uniformly Lipschitz functions with fixed range whose domains were converging in the intrinsicflat sense, and one for sequences of uniformly local isometries betweenspaces which were converging in the intrinsic flat sense. A basic BolzanoWeierstrauss Theorem was proven for sequences of points in such sequencesof spaces. In addition, it was proven that when a sequence of manifolds hasa pre-compact intrinsic flat limit then the metric completion of the limitwas the Gromov-Hausdorff limit of regions within those manifolds. Applications and suggestedapplications of these results were described in thefinal section of this paper.One of the most powerful theorems in metric geometry was theArzela-Ascoli Theorem which provides a continuous limit for sequencesof equi-continuous functions between two compact spaces. This theorem has been extended by Gromov and Grove-Petersen to sequences offunctions with varying domains and ranges where the domains and theranges respectively converge in the Gromov-Hausdorff sense to compactlimit spaces. However, such a powerful theorem does not hold when thedomains and ranges only converge in the intrinsic flat sense due to thepossible disappearance of points in the limit.

Mateusz Krukowski*et al.,* [27] have presented the Arzela-Ascoli theorem in the setting of uniformspaces. At first, recall well-known facts and theorems coming from monographs of Kelley and Willard. The main part of the paper introduces the notionof extension property which, similarly as equicontinuity, equates different topologies on $C(X,Y)$. This property enables us to prove the Arzelà-Ascoli theoremfor uniform convergence. The paper culminates with applications, which were motivated by Schwartz's distribution theoryUsing the Banach-Alaoglu-Bourbakitheorem, establish relative compactness of subfamily.

Cristina Antonescu*et al.,* [28] have presented arelaxation in the way a length function was usedin the construction of a metric, andthenshow that for groups of rapiddecay there were many metrics relatedto a length function whichhave all the expectedproperties. At the endshow that this notion allows a noncommutative version of the Arzela–Ascoli theorem.On a discrete group G; a length function may implement a spectral triple on the reduced group C-algebra. Following A. Connes, the Dirac operator of the triple then can induce ametric on the state space of the reduced group C-algebra.

2.6. Review related on diffusion equations

Xiao-Jun Yang *et al.,* [29] have presented the general fractional-order diffusion equations within thenegative Prabhakar kernel were considered for the first time. With the aid ofthe Laplace transform, the series solutions for the problems with the generalMittag-Leffler functions were discussed in detail. The obtained results were efficientin the description of the anomalous behaviors of the diffusive process. Fractional-order calculus operators have successfully applied to model the linear and nonlinear problems involvingthe power-law phenomena in mathematical physics, such as the rheological models with power law, anomalousrelaxation in dielectrics,diffusive models within the Mittag-Leffler noise,diffusion-wave with scale-invariantsolutions,and others.

NdolaneSene*et al.,* [30] have presented two types of diffusion processes obtained with the fractional diffusion equations described by the AtanganaBaleanu-Caputo (ABC)fractional derivative. The mean square displacement (MSD) concept has beenused to discuss the types of diffusion processes obtained when the order of thefractional derivative takes certain values. Many types of diffusion processes existand depend to the value of the order of the used fractional derivatives: the fractional diffusion equation with the subdiffusive process, the fractional diffusionequation with the superdiffusive process, the fractional diffusion equation withthe ballistic diffusive process and the fractional diffusion equation with the hyper diffusive process. Here use the Atangana-Baleanu fractional derivativeand analyze the subdiffusion process obtained when the order of ABC α was into(0, 1) and the normal diffusion obtained in the limiting case α = 1. The

Laplacetransform of the Atangana-Baleanu-Caputo fractional derivative has been usedfor getting the mean square displacement of the fractional diffusion equation.The central limit theorem has been discussed too, and the main results illustrated graphically.

Mohammed Al-Refai*et al.,* [31] have presented linear and nonlinear fractional diffusion equations with theCaputo fractional derivative of non-singular kernel that has been launched recently. Here, first derivesimple and strong maximum principles for the linear fractional equation. Here, thenimplement these principles to establish uniqueness and stability results for the linearand nonlinear fractional diffusion problems and to obtain a norm estimate of thesolution. In contrast with the previous results of the fractional diffusion equations, theobtained maximum principles were analogous to the ones with the Caputo fractionalderivative; however, extra necessary conditions for the existence of a solution of thelinear and nonlinear fractional diffusion models were imposed. These conditions affectthe norm estimate of the solution as well.

Xiao-Jun Yang *et al.,* [32] have developed the anomalous diffusion equations involving the general derivatives with the decay exponential kernel for the first time. With the useof the Laplace transform, the analytical solutions with graphs were discussed indetail. The results were accurate and efficient in the description of the behaviorsof the anomalous diffusion problems.The analytical solutions with graphs were also presented in detail. The results were useful to describe theanomalous diffusion problems with the GDs involving the decay exponential kernel.

Hijaz Ahmad *et al.,* [33] have presented modified versions of variational iteration algorithms for the numericalsimulation of the diffusion of oil pollutions. Three numerical examples were given to demonstrate the applicability andvalidity of the proposed algorithms. The obtained results were compared with the existing solutions, which reveal thatthe proposed methods were very effective and can be used for other nonlinear initial value problems arising in scienceand engineering.Based on the results, it has been found that the appliedmethods will be able to use without using discretization, shape parameter, Adomian polynomials, transformation,linearization or restrictive assumptions and thus

were particularly perfect with the expanded and flexible nature of thephysical problems and can be easily extended to fractal calculus arise in ocean engineering and science.

2.7. Review related on Adomian decomposition method

Dumitru Baleanu *et al.,* [34] have presented a fractional-order epidemicmodel for childhood diseases with the new fractional derivative approach proposed by Caputo and Fabrizio. By applying the Laplace Adomian decomposition method (LADM), we solve the problem and the solutions were presented as infinite series converging to the solution. Here, prove the existence, uniqueness, and stability of the solution by using the fixed-point theory. Also, provide some numerical results to illustrate the effectiveness of the new derivative.

Rasool Shah *et al.,* [35] have presented the analytical solution of the fractional-order dispersivepartial differential equations, using the Laplace–Adomian decomposition method. The Caputooperator was used to define the derivative of fractional-order. Laplace–Adomian decompositionmethod solutions for both fractional and integer orders were obtained in series form, showing higherconvergence of the proposed method. Illustrative examples were considered to confirm the validity ofthe present method. The fractional order solutions that were convergent to integer order solutions were also investigated.

Wenjin Li *et al.,* [36] have presented the Adomian decomposition method (ADM for short)including its iterative scheme and convergence analysis, which was a simple andeffective technique in dealing with some nonlinear problems. Here, take algebraicequations and fractional differential equations as applications to illustrate ADM'sefficiency. By applying the ADM, one can constructapproximate solutions to algebraic equations, fractional ordinary differential equations(time-fractional Riccati equations etc.), fractional partial differential equations (timefractional Kawahara equations, modified time-fractional Kawahara equations etc.), and even integro-differential equations, differential algebraic equations and so on. In practicalapplications, can take a finite sum according to the accuracy need.

Ahmed A hamoud*et al.,* [37] have presented a combined form for solvingnonlinear interval Volterra-Fredholm integral equations of the second kind based on the modifying Laplace Adomian decompositionmethod. Here, find the exact solutions of nonlinear interval VolterraFredholm integral equations with less computation as compared withstandard decomposition method. Finally, an illustrative example hasbeen solved to show the efficiency of the proposed method.

Shahid Mahmood *et al.,* [38] have developed hybrid method called Laplace Adomian Decomposition Method(LADM)for the analytical solution of the system of time fractional Navier-Stokes equation.The solution of this system can be obtained with the help of Maple software, which provide LADMalgorithm for the given problem. Moreover, the results of the proposed method were compared withthe exact solution of the problems, which has confirmed, that as the terms of the series increases theapproximate solutions were convergent to the exact solution of each problem. The accuracy of themethod was examined with help of some examples. The LADM, results have shown that, the proposedmethod has higher rate of convergence as compare to ADM and HPM.

Fazal Haq*et al.,* [39] have developed Numerical solution of fractional order smokingmodel via laplace Adomian decomposition method. Here, study analyticalsolution (approximate solution) of the concerned model with the help of Laplace transformation.The solution of the model will be obtained in form of infinite series which converges rapidly toits exact value. Moreover, compare results with the results obtained by Runge-Kuttamethod. Some plots were presented to show the reliability and simplicity of the method.

2.8. Review related on fixed point theorem applications

Tamer Nabil *et al.,* [40] have developed a new version of Kransnoselskii fixedpoint theorem. This generalization was more general becausecan study the N-tupled fixed point by it. Another advantage of the proposed N-tupled fixed point was to study theexistence of systems of more than two operator equations. For that, applied the abstract proposed fixed-point theorem toprove the existence of solution of the system of

N-RL- FDEs. Here, gave an example to illustrate the abstract proposed fixed-point theorem.

Muhammad Iqbal *et al.,* [41] have developed to establish conditions for obtaining mild solutions to a coupled systems of multipoint boundary value problems (BVPs) offractional order hybrid differential equations (FHDEs) with nonlinear perturbations of second type. In the concerned problem, consider a proportionaltype delay that represent a famous class of differential equation called pantograph equations. Hence, establish sufficient conditions for at least onemild solution to the coupled system of fractional hybrid pantograph differentialequations (FHPDEs) by using Burton and couple-type fixed-point theorems. Here also provide examples to show the applicability of results.

KhanitinMuangchoo-in *et al.,* [42] have presented a strategy based on fixed point iterative methods tosolve a nonlinear dynamical problem in a form of Green's function with boundaryvalue problems. First, the authors construct the sequence named Green's normal-Siteration to show that the sequence converges strongly to a fixed point, this sequencewas constructed based on the kinetics of the amperometric enzyme problem. Finally,the authors show numerical examples to analyze the solution of that problem.

Lucas Wangwe*et al.,* [43] have presented a common fixed-point theorem for F-Kannan mappings in metric spaces with an application tointegral equations. The main result of the paper will extend and generalize the recent existing fixed-point results in the literature. And, also provided illustrative examples and some applications to integral equation, nonlinear fractional differential equation, andordinary differential equation for damped forced oscillations to support the results.

Amjad Shaikh *et al.,* [44] have presented fractional differential equations including Caputo–Fabriziodifferential operator. The conditions for existence and uniqueness of solutions offractional initial value problems was established using fixed point theorem andcontraction principle, respectively. As an application, the iterative Laplace transformmethod (ILTM) was used to get an approximate solution for nonlinear fractionalreaction–diffusion equations, namely the Fitzhugh–Nagumo equation and the Fisherequation in the Caputo–Fabrizio sense. The obtained approximate solutions

werecompared with other available solutions from existing methods by using graphicalrepresentations and numerical computations. The results reveal that the proposedmethod was most suitable in terms of computational cost efficiency, and accuracywhich can be applied to find solutions of nonlinear fractional reaction–diffusionequations.

3. Conclusion

Nowadays, the mathematical models involving fractional order derivative were given noticeable importance because they were more accurate and realistic as compared to theclassical order models. Motivated by the advancement of fractional calculus, many researchers have focused to investigate the solutions of nonlinear differentialequations with the fractional operator by developing quite a few analytical or numericaltechniques to find approximate solutions. These differential equations involve several fractional differential operators like Riemann–Liouville, Caputo, Hilfer etc.However, these operators possess a power law kernel and have limitations in modelingphysical problems. To overcome this difficulty, recently an alternate fractional differentialoperator having a kernel with exponential decay has been introduced by Caputo and Fabrizio. In this chapter, the recent related works of diffusion equations, fixed point theorems, Adomian decomposition method and fixed-point theorem applications. From the review, the Banach fixed point theorems and Adomian decomposition method will be provided efficient results to solve the nonlinear mathematical problems. Hence, based on fixed point theorem and Adomian decomposition method will be developed to solve the nonlinear mathematical problems.

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