On Fundamental Attributes on Homomorphism of μ-anti- Fuzzy Subgroups

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Abstract. In this paper, we initiate the study of the notion of μ-anti- FSG defined on fuzzy set and probe things that each anti-FSG is μ-anti-FSG. We also prove that the merchandise of two μ-anti-FSGs is μ-anti-FSG. Additionally, we investigate the effect on the image and inverse image of μ-anti- FSG under group homomorphism and establish an isomorphism between the quotient group regard to μ-anti-FSG and quotient group with regard to the normal subgroup H_{A_μ}.

Key words: Fuzzy set (FS); Fuzzy Subset (FSb); Anti-fuzzy subgroup (Anti-FSG); μ -Anti-fuzzy set (μ-Anti-FS); μ-Anti-fuzzy subgroup (μ -Anti-FSG).
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1. Introduction

Zadeh [23] launched the study of FSs in 1965. Later on, Rosenfeld [20] invented the theory of fuzzy groups by using the idea of FSs in 1971. The idea of level subgroups of fuzzy group was innovated by Das [8] in 1981. Liu [13] described the fuzzy invariant subgroups in 1982. Mukherjee et.al. [18] introduced the concept of FCSs in 1984. Bhattacharya [4] explored the considerable characterizations of FSG in 1987. Choudhury et al. [6] acquainted the persuasion of fuzzy homomorphism between two groups and investigated its effect on FSGs in 1988. Akgul [3] meditated the notion of level subgroups of FNSGs and their homomorphisms in 1988. Mashour et al. [14] discussed many important properties of FNSGs in 1990. Biswas [5] commenced the opinion of anti-FSG in 1990. Dixit, et al. [9] studied the union of FSGs in 1990. Gupta

[11] developed many classical t-operators in 1991. Kumar et al. [12] explored the FNSG, fuzzy direct product and fuzzy quotients in 1992. Malik et al. [15] investigated the normality of FSGs in 1992. Filep [10] established the structure and construction of FSGs of group in 1992. Chakraborty and Khare [7] examined the behavior of the composition of fuzzy homomorphism and proved the fundamental theorem of homomorphism in 1993. Asaad and Zaid [2] proposed the study of FSGs of nilpotent groups in 1993. Ajmal [1] described the homomorphism of FSGs, fuzzy quotient groups and correspondence theorem in 1994. Morsi et.al.[17] examined fuzzy quotient groups and level structures in 1994. Mishref [16] described the normal, subnormal and composition series of FNSGs in 1995. Ray [19] developed key features of the product of two FSb and FSGs in 1999. Sharma [21] expounded α -anti FSGs and depicted their several algebraic properties in 2012. Moreover, fundamental algebraic attributes of α-fuzzy subgroups were established by the same author [22] in 2013. This paper is formed because the section 2 contains the fundamental definitions of anti-FSG and related results which are thoroughly crucial to know the novelty of this paper. In section 3, we clarify an μ-anti-FS with regard to t-conorm. In section 4, we use the classical group homomorphism to research the behavior of homomorphic image (inverse-image) of μ-anti-FSG.

2. Preliminaries

Definition 2.1. [23]: A FS A of a nonempty set P is a function $A : P \to [0, 1]$.

Definition 2.2. [3]: Let A be FSb of a group H. Then A is said to a FSG if $A(u^{-1}v) \geq \min\{A(u), A(v)\}$, for all $u, v \in H$.

Definition 2.3. [5]: Let A be FSb of a group H. Then A is said to an anti-FSG if $A(u^{-1}v) \leq \max\{A(u), A(v)\}$, for all $u, v \in H$

Definition 2.4. [11]: A function $t : [0, 1] \times [0, 1] \in [0, 1]$ is said to be a t-conorm on $[0, 1]$ if and only if t admits following properties for all $u_1, u_2, u_3, u_4 \in [0, 1]$.

(1) $t(u_1, u_2) = t(u_2, u_1)$
(2) $t(u_1, t(u_2, u_3)) = t(t(u_1, u_2), u_3)$
(3) $t(u_1, 1) = t(1, u_1) = 1$
(4) $If u_1 \leq u_3$ and $u_2 \leq u_4$ then $t(u_1, u_2) \leq t(u_3, u_4)$

Definition 2.5. [11]: Let $S_p : [0,1] \times [0,1] \to [0,1]$ be the algebraic sum t-conorm on $[0,1]$ defined by $S_p\{u_1, u_2\} = u_1 + u_2 - u_1u_2, 0 \leq u_1 \leq 1, 0 \leq u_2 \leq 1$ Clearly algebraic sum t-conorm admits the axioms of t-conorm.

3. Algebraic Attributes on μ-anti-FSbs, μ-Anti-FSG and their characteristics

Definition 3.1. : Let P be a non empty sets and A be a FSb of P and $\mu \in [0,1]$. Then the FSA is called the μ-Anti- FSb of P and defined by

$$A_\mu(m) = S_p\{A(m), 1-\mu\}, \textbf{for all} \;\; m \in P$$

Definition 3.2. Let H be a group and A be a FSb of H and $\mu \in [0,1]$. Then A is called μ-Anti-FSG of H if

(1) $A_\mu(mn) \leq \max\{A_\mu(m), A_\mu(n)\}$, for all $m, n \in H$
(2) $A_\mu(m^{-1}) = A_\mu(m)$.

Remark 3.3. *Clearly $A^1(m) = A(m)$ and $A^0(m) = 1$*

Lemma 3.4. *Let A and B be two arbitrary FSb of P. Then $(A \cup B)_\mu = A_\mu \cup B_\mu$*

Proof. Consider,

$$\begin{aligned}(A \cup B)_\mu(m) &= S_p\{(A \cup B)(m), 1-\mu\} \\ &= S_p\{max\{A(m), B(m)\}, 1-\mu\} = max\{S_p\{A(m), 1-\mu\}, S_p\{B(m), 1-\mu\}\} \\ &= max\{A_\mu(m), B_\mu(m)\} = (A_\mu \cup B_\mu)(m), \quad forall \;\; m \in P\end{aligned}$$

This implies that $(A \cup B)_\mu = A_\mu \cup B_\mu$ □

Theorem 3.5. *Let $f : M \to N$ and A and B be two FSbs of M and N respectively, then*

(i) $f^{-1}(B_\mu)(m) = (f^{-1}(B))_\mu(m)$, *for all* $m \in M$
(ii) $f(A_\mu)(n) = (f(A))_\mu(n)$, *for all* $n \in N$

Proof. (i) $f^{-1}(B_\mu)(m) = B_\mu(f(m)) = S_p\{B(f(m)), 1-\mu\} = S_p\{f^{-1}(B)(m), 1-\mu\}$

$$f^{-1}(B_\mu)(m) = (f^{-1}(B))_\mu(m), \text{ for all } m \in M$$

(ii) $f(A_\mu)(n) = sup\{A_\mu(m) : f(m) = n\}$
$= sup\{S_p\{A(m), 1-\mu\} : f(m) = n\}$
$= S_p\{sup\{\{A(m) : f(m) = n\}, 1-\mu\}\}$
$= S_p\{f(A)(n), 1-\mu\} = (f(A))_\mu(n)$, for all $n \in N$

Hence, $f(A_\mu)(n) = (f(A))_\mu(n)$ □

4. Fundamental Attributes on Homomorphism of μ-anti- FSGs

Theorem 4.1. *Let $f : H_1 \to H_2$ be a bijective homomorphism from a group H_1 to a group $H_2$and A be aμ-Anti-FSG of group H_1. Then $f(A)$ is a μ-anti-FSG of group H_2.*

Proof. Let A be a μ-Anti-FSG of group H_1. Let $n_1, n_2 \in H_2$ be any element. Then there exists unique elements $m_1, m_2 \in H_1$ S.t $f(m_1) = n_1$and $f(m_2) = n_2$.
Consider $(f(A))^\mu(n_1n_2) = S_p\{f(A)(n_1n_2), 1-\mu\} = S_p\{f(A)(f(m_1)f(m_2)), 1-\mu\} = S_p\{f(A)(f(m_1m_2)), 1-\mu\} = S_p\{A(m_1m_2), 1-\mu\} = A_\mu(m_1m_2)$
$\leq \max\{A_\mu(m_1), A_\mu(m_2)\}, forall m_1, m_2 \in H_1 such that f(m_1) = n_1 and \quad f(m_2) = n_2$
$\leq \max\{supA_\mu(m_1) : f(m_1) = n_1, sup\{A_\mu(m_2) : f(m_2) = n_2\}\}$
$= \max\{f(A_\mu)(n_1), f(A_\mu)(n_2)\} = \max\{(f(A))_\mu(n_1), (f(A))_\mu(n_2)\}$
Thus, $(f(A))_\mu(n_1n_2) \leq \max\{(f(A))_\mu(n_1), (f(A))_\mu(n_2)\}$.
Further, $(f(A))_\mu(n^{-1}) = f(A)_\mu(n^{-1}) = sup\{A_\mu(m^{-1}) : f(m^{-1}) = n^{-1}\}$
$= sup\{A_\mu(m) : f(m) = n\} = f(A_\mu)(n) = (f(A))_\mu(n)$.
Consequently,$f(A)$ is μ-Anti-FSG of H_2. □

Theorem 4.2. *Let $f : H_1 \to H_2$ be a homomorphism from group H_1 into a group H_2 and B be a μ-Anti-FSG of group H_2. Then $f^{-1}(B)$ is μ-Anti-FSG of group H_1.*

Proof. Let B be μ-Anti-FSG of group H_2. Let $m_1, m_2 \in H_1$ be any elements, then

$$(f^{-1}(B))_\mu(m_1m_2) = f^{-1}(B_\mu)(m_1m_2) = B_\mu(f(m_1m_2)) = B_\mu(f(m_1)f(m_2))$$
$$\leq \max\{B_\mu(f(m_1)), B_\mu(f(m_2))\} = \max\{f^{-1}(B_\mu)(m_1), f^{-1}(B_\mu)(m_2)\}$$
$$= \max\{(f^{-1}(B))_\mu(m_1), (f^{-1}(B))_\mu(m_2)\}$$

Thus, $(f^{-1}(B))_\mu(m_1m_2) \leq \max\{(f^{-1}(B))_\mu(m_1), (f^{-1}(B))_\mu(m_2)\}$.
Further, $(f^{-1}(B))_\mu(m^{-1}) = f^{-1}(B_\mu)(m^{-1}) = B_\mu(f(m^{-1})) = B_\mu(f(m)^{-1})$

$= B_\mu(f(m)) = f^{-1}(B_\mu)(m)$

Hence, $(f^{-1}(B))_\mu(m^{-1}) = f^{-1}(B_\mu)(m)$.

Consequently, $f^{-1}(B)$ is μ-Anti-FSG of a group H_1. □

5. Conclusions

In present work, the ideas of μ-Anti-FS, μ-anti-FSG and μ-Anti-FCS of a given group are delineated. Additionally , we have extended the study of this ideology to research the effect of image and inverse image ofμ-Anti-FSG under group homomorphism.

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