## **Reverse Super Edge Magic Strength of Some Graphs**

Kotte Amaranadha Reddy<sup>1</sup> and S Sharief Basha<sup>2</sup> <sup>1</sup>Research Scholar, Department of Mathematics, VIT, Vellore. <sup>2</sup>Assistant Professor (Sr.), Department of Mathematics, VIT, Vellore. Email-shariefbasha.s@gmail.com.

## Abstract:

If  $\exists$  a bijection f from  $V \cup E \rightarrow \{1,2,3,...,|V| + |E|\}$  as well as  $f(e) - \{f(u) + f(v)\} = k$  is constant for all  $e \in E$  and  $f(V) = \{1,2,3,...,|V|\}$ , Reverse super edge magic (RSEM) is a graph G(V, E). The smallest of all k, where the minimum is indicated by rsm(G), and it runs through all RSEM labelings of G, is the graph G's RSEM strength. The RSEM strength of some graph families can be found here.

Keywords: RSEM, RSEM Strength.

Mathematical Classifications: 05C78.

## 1. Introduction:

Avadayappan S et al. established the notion of graph magic strength (MS) in [1]. That is,  $m(G) = min\{c(f) : f \text{ is a magic labeling of } G\}$  and They have derived the MS of various graph families. The minimum of all c(f) is described as sm(G), where the minimum is described over all super edge-magic (SEM) labelings f of G. Avadayappan S et al. [2] introduced the super MS of a graph G, The minimum of all c(f) is described as sm(G), where the smallest is described in all super edge-magic (SEM) labelings f of G. In [3], Swaminathan V and Jeyanthi P calculated the SEM strength of a few graph families. Jeyanthi P and Selvagopal P discovered the k-super MS of all lengths can be considered as k-polygonal snakes and the H-super MS of a chain of any two-connected simple graph H. They also make a conjecture regarding the  $P_h$  – super MS of  $P_n$  for  $2 \le h \le n$  [4]. Sharief Basha S and Madhusudhan Reddy K showed in [5] that few various festoon trees have reverse super edge-magic (RSEM) strength. Faraha Ashraf et al. introduced a graph's total H-irregularity strength, premised on this parameter, also demonstrated the accurate values of this parameter for some families of graphs [6]. In [7], R. Ichishima, exhibited a connection between the super MS and a particular type of strength, which leads us to sharp bounds for the super MS of SEM graphs. In [8], Mathew Varkey T. K and Mini. S. Thomas was the determined reverse process (RP) of graphoidal of a MS is called reverse graphoidal (RG) MS and proved RG MS of Parachute, Armed Crown graph. The RP of magic graphoidal strength is presence as RG MS and also proved to RG MS of Path, Star, Comb, and  $[P_n; S_1]$  determined by Mathew Varkey T.K and Mini.S.Thomas [9]. For any integers  $m, n \ge 3$ , I Nengah Suparta and I Gusti Putu Suharta proposed bounds for the join graph's edge irregularity strength  $P_m + \overline{K_n}$  in [10]. In [11], Yeni Susantia et al. mirror-staircase, double staircase, and staircase graphs determined the precise value of total edge irregularity strength. In [12], Rikio Ichishima et al. given formulas for such edge-strength of some graph classes whose line graphs are defined in terms of various graph operations.

We also take note of the following truth. let f be a SEM labelling of a valence k(p,q) –graph G, for all edge  $uv \in E(G)$ , then  $f(uv) - \{f(u) + f(v)\} = k$ . When all the constants acquired at each edge of G are added together, we get

$$qk = \sum_{u \in V(G)} f(uv) - \sum_{u \in V(G)} f(u)d(u)$$
<sup>(1)</sup>

**Lemma 1:** RSEM is a graph G with p vertices and q edges if a bijective function  $f: V(G) \rightarrow \{1,2,3,...,p\}$  contains q consecutive numbers in the set  $S = \{f(x) + f(y)/xy \in E(G)\}$  spreads to the graph G was RSEM labelling using the reverse magic constant k = p + q - s, which s = max(S) and

$$S = \{f(x) + f(y)/xy \in E(G)\} = \{(p+1) - k, (p+2) - k, (p+3) - k, \dots, (p+q) - k\}$$

**Theorem 1:** For each odd n,n > 7, the graph  $C_n$ , which is a cycle with a chord consisting of two vertices at a distance of 3, is RSEM.

**Proof:** Let's call the graph G and  $C_n$  s a chord consisting of two  $C_n (n \ge 7)$  vertices distance by 3.

Let  $V(G) = \{v_1, v_2, ..., v_n\}$  Next join as a chord for G, combine the vertices  $v_1$  and  $v_{n-2}$  so that  $(v_1, v_{n-}) = 3$ . G has n vertices and n + 1 edges, as shown. Define the vertex labelling  $f: V(G) \rightarrow \{1, 2, ..., n\}$  then

$$f(v_j) = \begin{cases} \frac{j+1}{2} & \text{if } j \text{ is odd} \\ \frac{n+j+1}{2} & \text{if } j \text{ is even} \end{cases}$$

Note that

 $S = \{f(x) + f(y): xy \in E(G)\} = \{\frac{n+1}{2}, \frac{n+3}{2}, \dots, \frac{3n+1}{2}\}$  is a collection of consecutive integers. As a result of Lemma 1, f extends to an RSEM labelling of G with valence  $k = p + q - s = \frac{n+1}{2}$ .

**Theorem 2:** For every odd  $n \ge 3$ ,  $rsm(C_n \odot P_2) = \frac{5n-1}{2}$ . **Proof.** For every odd  $n \ge 3$ ,  $rsm(C_n \odot P_2) = \frac{5n-1}{2}$ .

Let *f* be a RSEM labelling with valence *k* of the graph  $C_n \odot P_2$ . Then, by applying (1) to p = 3n and q = 4n we get

$$\begin{aligned} 4nk &= 2c \text{ Where } a_i, b_i(1 \le i \le n) \text{ are the adjacent } C_n \text{ vertices } v_i \text{ in } C_n \odot P_2. \text{ Again} \\ 4nk &= \sum_{e \in E} f(e) - \{2\sum_{i=1}^n f(a_i) + 2\sum_{i=1}^n f(b_i) + 4\sum_{i=1}^n f(v_i)\} \text{ where } a_i b_i(1 \le i \le n) \text{ are} \\ the vertices in  $C_n \odot P_2 \text{ that are adjacent to the rim vertices } v_i \text{ of } C_n. \\ 4nk &= 2\sum_{e \in E} f(e) - \{[\sum_{i=1}^n f(a_i) + \sum_{i=1}^n f(b_i) + \sum_{i=1}^n f(v_i) + \sum_{e \in E} f(e)] + [\sum_{i=1}^n f(a_i) + \sum_{i=1}^n f(b_i) + \sum_{i=1}^n f(v_i)] + 2\sum_{i=1}^n f(v_i)\} \\ &= 2[(3n+1) + (3n+2) + \dots + 7n] - \{[1+2+\dots+7n] + [1+2+\dots+3n] + 2\sum_{i=1}^n f(v_i)\} \\ \text{Hence, } k &= 10n + 1 - \{\frac{7n(7n+1)}{2} + \frac{3n(3n+1)}{2} + \frac{1}{2n}\sum_{i=1}^n f(v_i)\} \\ &\geq 10n + 1 - \{\frac{7n(7n+1)}{2(4n)} + \frac{3n(3n+1)}{2n} + \frac{1}{2n}[1+2+\dots+n]\} \\ &= 10n + 1 - \{\frac{58n+10}{8} + \frac{1}{2n}\frac{n(n+1)}{2}\} \end{aligned}$$$

$$= 10n + 1 - \{\frac{29n+5}{4} + \frac{n+1}{4}\}$$
$$= \frac{5n-1}{2}$$
i.e.,  $k \ge \frac{5n-1}{2}$ 

Thus, For every odd  $n \ge 3$ ,  $rsm(C_n \odot P_2) = \frac{5n-1}{2}$ .

Therefore, For every odd  $n \ge 3$ ,  $rsm(C_n \odot P_2) = \frac{5n-1}{2}$ . **Theorem 3:** For every odd  $n \ge 3$ ,  $\frac{8n-1}{2} \le rsm(C_n \odot P_3) \le \frac{9n-1}{2}$ . **Proof:** We known that  $C_n \odot P_3$  has 4n vertices and 6n edges.

In  $C_n \odot P_3$ , let  $a_i (1 \le i \le 2n)$  define the second degree vertices 2,  $b_i (1 \le i \le n)$  the third degree vertices, and  $V_i (1 \le i \le n)$  the fifth degree vertices.

Let *f* represent the RSEM labelling of the graph  $C_n \odot P_3$  with valence *k*. Then, applying (1) to p = 4n and q = 6n, we obtain

$$\begin{aligned} 6nk &= \sum_{e \in E} f(e) - \{2 \sum_{i=1}^{2n} f(a_i) + 3 \sum_{i=1}^{n} f(b_i) + 5 \sum_{i=1}^{n} f(v_i)\} \\ &= 2 \sum_{e \in E} f(e) - \{[\sum_{i=1}^{2n} f(a_i) + \sum_{i=1}^{n} f(b_i) + \sum_{i=1}^{n} f(v_i) + \sum_{e \in E} f(e)] + [\sum_{i=1}^{2n} f(a_i) + \sum_{i=1}^{n} f(b_i) + \sum_{i=1}^{n} f(v_i)] + \sum_{i=1}^{n} f(b_i) + 3 \sum_{i=1}^{n} f(v_i)\} \\ &= 2[(4n+1) + (4n+2) + \dots + 10n] - \{(1+2+\dots + 10n) + (1+2+\dots + 4n) + (1+2+\dots + n) + 3(1+2+\dots + n)\} \\ &= 2 \frac{6n(10n+4n+1)}{2} - \{\frac{10n(10n+1)}{2} + \frac{4n(4n+1)}{2} + \frac{n(n+1)}{2} + 3 \frac{n(n+1)}{2}\} \\ &= 14n + 1 - \{\frac{1}{6n} [5n(10n+1) + 2n(4n+1) + \frac{n(n+1)}{2} + 3 \frac{n(n+1)}{2}]\} \\ &= 14n + 1 - \{\frac{1}{6} [58n + 7 + \frac{n+1}{2} + \frac{3(n+1)}{2}]\} \\ &= 14n + 1 - \{\frac{1}{6} [\frac{116n+14+4n+4}{2}]\} \\ &= \frac{8n-1}{2} \end{aligned}$$

Thus, For every odd  $n \ge 3$ ,  $rsm(C_n \odot P_3) \ge \frac{8n-1}{2}$ . Therefore, Thus, For every odd  $n \ge 3$ ,  $rsm(C_n \odot P_3) \ge \frac{8n-1}{2}$ .

**Theorem 4:** Let *G* denote the graph formed by an odd cycle  $C_n (n \ge 7)$  with a chord connecting two vertices at a distance of 3. Then, for all odd  $n \ge 7$ ,  $rsm(G) = \frac{5n+3}{2}$ .

**Proof:** Let  $V(G) = \{v_1, v_2, \dots, v_n\}$ . Then, using theorem 1, connect the vertices vertices  $v_i$  and  $v_{n-2}$  to form a chord for G,  $rsm(G) \le \frac{n+1}{2}$  for every  $n \ge 7$ .

Let  $V(G) = \{v_1, v_2, \dots, v_n\}$ . By theorem 1, join the vertices  $v_i$  and  $v_{n-2}$  as a chord for G, for all odd  $n \ge 7$ ,  $rsm(G) = \frac{5n+3}{2}$ .

Assume f is a RSEM labelling of the graph G with valence k. Then, applying (1) to p = n and q = n + 1, we get

$$\begin{array}{l} (n+1)\tilde{k} &= \sum_{e \in E(G)} f(e) - \{2\sum_{\substack{i=2 \\ i \neq (n-2)}}^{n} f(v_i) + 3f(v_1) + 3f(v_{n-2})\} \\ &= 2\sum_{e \in E(G)} f(e) - \{\left[\sum_{i=1}^{n} f(v_i) + \sum_{e \in E(G)} f(e)\right] + \sum_{i=1}^{n} f(v_1) + f(v_1) + f(v_{n-2})\} \end{array}$$

Thus, for odd n,  $rsm(G) = \frac{5n-3}{2}$ . Therefore, for odd n,  $rsm(G) = \frac{5n-3}{2}$ .

**Theorem 6:** If *m* is odd Then 3-star  $S_{n,3}$  is RSEM. Then, for odd n, rsm(G) = 2n - 1. **Proof:** Let *n* is odd. Assume 3n + 1 be the degree of vertex *x* in  $S_{n,3}$  and 3 is the length of  $i^{th}$  path of  $x u_i v_i w_i$  for  $1 \le i \le m$ .

The paths are RSEM and since  $S_{1,3} \cong P_4$ , when m = 1 the out come is true. Assume that m is an odd number and n > 3 with 3n + 1 vertices and 3n edges.

We have 
$$qk = \sum_{u \in V(G)} f(uv) - \sum_{u \in V(G)} f(u)d(u)$$
  
 $(3n)k = \{(3n + 2) + (3n + 3) + \dots + (6n + 1)\} - \{5(n + 1) + 2\sum_{u \in V} f(u)d(u) - \sum_{\sum_{i \leq n} f(w_i)}\}$   
We have  $qk = \sum_{u \in V(G)} f(uv) - \sum_{u \in V(G)} f(u)d(u)$   
 $(3n)k = \{(3n + 2) + (3n + 3) + \dots + (6n + 1)\} - \{5(n + 1) + 2\sum_{u \in V} f(u)d(u) - \sum_{\sum_{i \leq n} f(w_i)}\}$   
 $(3n)k = \{(3n + 2) + (3n + 3) + \dots + (6n + 1)\} - \{n(n + 1) + 2[1 + 2 + 3 + \dots + (3n + 1)] - [(2n + 1) + (\frac{5n + 1}{2}) + (\frac{5n - 1}{2}) + \dots + (2n + 2)] + (3n + (3n - 1) + \dots + \frac{5n + 3}{2})\}$   
 $(3n)k = \frac{3n}{2}(9n + 3) - \{n(n + 1) + 2(\frac{2n + 1}{2})(3n + 2) + 2(\frac{2n}{2})(4n + 3) - [(2n + 1) + \frac{n - 1}{4}(2n + 2 + \frac{5n + 1}{2})] + \frac{n - 1}{4}(3n + \frac{5n + 3}{2})\}$   
 $(3n)k = \frac{3n}{2}(9n + 3) - \{n(n + 1) + 2(\frac{2n + 1}{2})(3n + 2) + 2(\frac{2n}{2})(4n + 3) - [(2n + 1) + \frac{n - 1}{4}(2n + 2 + \frac{5n + 1}{2})] + \frac{n - 1}{4}(3n + \frac{5n + 3}{2})\}$   
 $(3n)k = \frac{3n}{2}(9n + 3) - \{n(n + 1) + 2(\frac{2n + 1}{8})^2 + 6n - [2n + 1 + \frac{9n^2 - 4n - 5}{8} - \frac{1n^2 - 8n - 3}{8}\}$   
 $(3n)k = \frac{3n}{2}(9n + 3) - \{2n^2 + 2n + 8n^2 + 6n - [2n + 1 + \frac{9n^2 - 4n - 5}{8} - \frac{1n^2 - 8n - 3}{8}\}$   
 $(3n)k = \frac{3n}{2}(9n + 3) - \{2n^2 + 6n + \frac{20n^2 + 4n}{8}\}$   
 $(3n)k = \frac{3n}{2}(9n + 3) - \{6nn^2 + 6nn - \frac{20n^2 + 4n}{8}\}$   
 $(3n)k = \frac{3n}{2}(9n + 3) - \{6nn^2 + 6nn - \frac{20n^2 + 4n}{8}\}$   
 $(3n)k = \frac{3n}{2}(9n + 3) - \{6nn^2 + 6nn - \frac{20n^2 + 4n}{8}\}$   
 $(3n)k = \frac{3n}{2}(9n + 3) - \{6nn^2 + 6nn - 20n^2 - 4n\}$   
 $(3n)k = \frac{3n}{2}(9n + 3) - \frac{6nn^2 + 6nn - 20n^2 - 4n}{8}$   
 $(3n)k = \frac{3n}{2}(9n + 3) - \frac{6nn^2 + 6nn - 20n^2 - 4n}{8}$   
 $(3n)k = \frac{3n}{2}(9n + 3) - \frac{6nn^2 + 6nn - 20n^2 - 4n}{8}$   
 $(3n)k = \frac{3n}{2}(9n + 3) - \frac{6nn^2 + 6nn - 20n^2 - 4n}{8}$   
 $(3n)k = \frac{3n}{2}(9n + 3) - \frac{6nn^2 + 6nn - 20n^2 - 4n}{8}$   
 $(3n)k = \frac{3n}{2}(9n + 3) - \frac{6nn^2 + 6nn - 20n}{8}$   
 $(3n)k = \frac{3n}{2}(9n + 3) - \frac{6nn^2 + 6nn - 20n}{8}$   
 $(3n)k = \frac{3n}{2}(9n + 3) - \frac{6nn^2 + 6nn - 20n}{8}$   
 $(3n)k = \frac{3n}{2}(9n + 3) - \frac{6nn^2 + 6nn - 20n}{8}$   
 $(3n)k = \frac{3n}{2}(9n + 3) - \frac{6nn^2 + 6nn - 20n}{8}$   
 $(3n)k = \frac{3n}{2}(9n + 3) - \frac{6nn^2 + 6nn - 20n}{8}$   
 $(3n)k = \frac{3n}{2}$ 

$$\begin{aligned} (3n-1)k &= \frac{(3n-1)}{2}(9n) - \left\{ \left[ \frac{21n^2+7n}{2} \right] - \left[ \frac{6n^2+14n+4}{4} \right] \right\} \\ k &= \frac{3n-2}{2} \end{aligned}$$
Thus, for even  $n, rsm(G) &= \frac{3n-2}{2}.$   
Therefore, for even  $n, rsm(G) = \frac{3n-2}{2}.$   
**Case 2:** If  $n$  is odd  
 $qk &= \sum_{u \in V(G)} f(uv) - \left\{ 3\sum_{i=1}^{n} f(u_i) - 2\sum_{i=1}^{n} f(a_i) - \sum_{i=1}^{n} f(b_i) - f(c_i) - f(c_n) \right\}$   
 $(3n-1)k &= \left\{ (3n+1) + (3n+2) + \dots + (6n-1) \right\} - \left\{ 3(1+2+\dots+3n) - 2[3n+(3n-1) + (3n-2) + \dots + (2n+1)] - \left[ (1+3+5+\dots+(2n-1)) + (n+1) + 2n \right] \right\}$   
 $&= \left( \frac{3n-1}{2} \right) (3n+1+6n-1) - \left\{ 3\frac{3n}{2} (3n+1) - 2\frac{n}{2} (3n+2n+1) - \frac{n}{2} (2(1) + (n-1)2) - 3n-1 \right\}$   
 $&= \left( \frac{3n-1}{2} \right) (9n) - \left\{ \frac{27n^2+9n}{2} - 5n^2 - n - n^2 - 3n - 1 \right\}$   
 $&= \left( \frac{3n-1}{2} \right) (9n) - \left\{ \frac{27n^2+9n}{2} - 6n^2 - 4n - 1 \right\}$   
 $&= \left( \frac{3n-1}{2} \right) (9n) - \left\{ \frac{15n^2+n-2}{2} \right\}$   
 $(3n-1)k = \left( \frac{3n-1}{2} \right) (9n) - \frac{(15n^2+n-2)}{2}$   
 $k = \frac{9n}{2} - \frac{(5n+2)}{2}$   
 $k = \frac{4n-2}{2}$   
 $k = 2n-1$ 

Thus, for odd n, rsm(G) = 2n - 1. Therefore, for odd n, rsm(G) = 2n - 1.

**Theorem 8:** The total graph  $T(P_n)$  is RSEM for all *n*. Then, for all  $n \operatorname{rsm}(G) = 2n - 3$ . **Proof:** Let  $P_n$  be the path  $u_1, u_2, ..., u_n$  and  $e_j$  be the edge  $u_j, u_{j+1}$  for  $1 \le j \le (n-1)$ . Then the vertex and edge set of  $T(P_n)$  as denoted as

$$V(T(P_n)) = \{u_j, e_j: 1 \le i \le n, 1 \le j \le (n-1)$$
Note that  $T(P_n)$  has  $2n - 1$  vertices and  $4n - 5$  edges. Then  $q = 2p - 3$ .  
We have
 $qk = \sum_{u \in V(G)} f(uv) - \sum_{u \in V(G)} f(u)d(u)$ 
 $(4n - 5)k = \{2n + (2n + 1) + \dots + (6n - 6)\} - \{4(1 + 2 + \dots + (2n - 1) - 4n - 2n\}$ 
 $= \frac{(4n - 5)}{2}(2n + 6n - 6) - \{4(\frac{2n - 1}{2})(1 + 2n - 1) - 4n - 2n\}$ 
 $= \frac{(4n - 5)}{2}(2n + 6n - 6) - \{4(\frac{2n - 1}{2})(1 + 2n - 1) - 4n - 2n\}$ 
 $= \frac{(4n - 5)}{2}(8n - 6) - \{(4n - 2)(2n) - 6n\}$ 
 $= \frac{(4n - 5)}{2}(8n - 6) - (2n)(4n - 2 - 3)$ 
 $(4n - 5)k = (4n - 5)(4n - 3) - (2n)(4n - 5)$ 
 $k = (4n - 3) - 2n$ 
 $k = 2n - 3$ 
Thus, for all  $n rsm(G) = 2n - 3$ .

## **References:**

- [1] S. Avadayappan, P. Jeyanthi, R. Vasuki, Magic strength of a graph, Indian J. Pure Appl. Math. 31 (7) (2000) 873–883.
- [2] S. Avadayappan, P. Jeyanthi, R. Vasuki, Super magic strength of a graph, Indian J. Pure Appl. Math. 32 (11) (2001) 1621–1630.
- [3] V. Swaminathana, P. Jeyanthib, Super edge-magic strength of fire crackers, banana trees and unicyclic graphs, Discrete Mathematics 306 (2006) 1624 1636.
- [4] P. JEYANTHI and P. SELVAGOPAL, H-Supermagic Strength of Some Graphs, TOKYO J. MATH. VOL. 33, NO. 2(2010) 499-507.
- [5] S. Sharief Basha K. Madhusudhan Reddy, REVERSE MAGIC STRENGTH OF FESTOON TREES, italian journal of pure and applied mathematics 33 (2014) 191-200.
- [6] Faraha Ashraf et al., On H-Irregularity Strength of Graphs, Discussiones Mathematicae Graph Theory 37 (2017) 1067–1078.
- [7] R. Ichishima et al., Bounds for the strength of graphs, Australasian Journal of Combinatorics, Volume 72(3) (2018) 492–508.
- [8] Mathew Varkey T.K Mini.S.Thomas2, Circle related reverse-graphoidal Magic strength, International Journal for Research in Engineering Application & Management Vol-04(02) 426-430.
- [9] Mathew Varkey T.K 1 Mini.S.Thomas2, Reverse Graphoidal Magic Strength, IJCRT, Volume 6(2) 348-353.
- [10] I Nengah Suparta, I Gusti Putu Suharta, A note on edge irregularity strength of some Graphs, Indonesian Journal of Combinatorics 4 (1) (2020) 10–20.
- [11] Yeni Susanti et al., On Total Edge Irregularity Strength of Staircase Graphs and Related Graphs, Iranian Journal of Mathematical Sciences and Informatics Vol. 15(1) (2020) 1-13.
- [12] Rikio Ichishima et al., The edge-strength of graphs, Discrete Math. Lett. 3 (2020) 44–49.