Geo Chromatic Number of Middle, Central and Total Graph for P_n and C_n

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Abstract: *In this paper, we have focused about the Geo Chromatic Number (GCN) $\chi_{gc}(G)$ and derived the result of Middle, Central and Total graphs for path and cycle.*

Keywords**: Geodetic, Geodetic number, Middle graph, Central graph, Total graph**

1. INTRODUCTION

The concepts of the GCN of graph were introduced by SB Samli and SR Chellathurai [11]. In this paper, we consider a graph to be connected, finite, simple where V(G) is Vertex set and E(G) is edge set [1,6]. The distance between two vertices n_1, n_2 contained in V(G) is the minimum size of $n_1 – n_2$ paths in G. An $n_1 – n_2$ path of the size $d_G(n_1, n_2)$ is known as geodesic. We indicate $I_G(n_1, n_2)$ as the set of Vertices which are lies inside some $n_1 – n_2$ geodesics of G. A Vertex is said to be lie on $n_1 – n_2$ geodetic if c is an inner Vertex of P. The bounded interval $I(n_1, n_2)$ includes n_1, n_2 and all Vertices falls on some $n_1 – n_2$ geodesic of G. Consider a non- empty set $S \subseteq V(G)$. For a set I(S) = $\bigcup_{n_1,n_2\in S} I(n_1,n_2)$. If G is connected graph, thus S is a geodetic set g(S) such that I(S) = V (G). The minimum cardinality S of G is known geodetic number defined by g(G). A j – Vertex coloring of G is an allotment of j colors to the Vertices of G. The coloring is proper if no two joining Vertices accept same color such that $\chi(G) = j$ is said to be j – chromatic, where j $\le$k a minimum cardinality of a chromatic number of G is known as chromatic set [3, 4, 5, 7, 10].

In this paper, we discussed about Geo Chromatic Number (GCN) [8, 9] for Middle, Central and Total graph [2, 12].

2. Main results

Using results [4], the following results are derived.

3.MIDDLE, CENTRAL AND TOTAL GRAPH OF PATH

Theorem 3.1: For any path P_n , n > 2. Then $\chi_{gc}[M(P_n)]$ = n.

Proof: Consider the vertex set $V(P_n) = \{v_i : 1 \le i \le n\}$ and $E(P_n) = \{ e_i = v_i v_{i+1} : 1 \le i \le n – 1\}$. By the concept of middle graph of the path P_n, all the edges of $E(P_n)$ is partitioned by the vertices $\{ v_i' : 1 \le i \le n – 1\}$ is $M(P_n)$. Here by the vertex and edges set of $M(P_n)$ is $V[M(P_n)] = \{vi : 1 \le i \le n\} \cup \{ v_i' : 1 \le i \le n – 1\}$, where as $\{ v_i' : 1 \le i \le n – 1\}$ is the vertices of $M(P_n)$ corresponding to the edges $\{ v_i v_{i+1} : 1 \le i \le n – 1\}$ of P_n and $E[M(P_n)] = \{v_i v_i' : 1 \le i \le n\} \cup \{ v_i' v_{i+1}' : 1 \le i \le n – 2\} \cup \{ v_i' v_{i+1} : 1 \le i \le n – 1\}$. Since the set of all vertices of P_n is the minimum geodetic set of $M(P_n)$. By allotting the proper color, the vertices in S belongs to the different color class, $\chi[M(P_n)]$ = 3. Thus, S_c is both geodetic and chromatic set of $M(P_n)$. Hence $\chi_{gc}[M(P_n)]$ = n.

Theorem 3.2: For any path P_n , n > 2. Then $\chi_{gc}[C(P_n)]$ = n.

Proof: Consider the path P_n has vertex set $\{v_i : 1 \leq i \leq n\}$ and the edge set $\{e_i = v_i v_{i+1} : 1 \leq i \leq n-1\}$. By the concept of central graph of the path P_n, Let us partition all the edge of $E(P_n)$ as $\{v_i' : 1 \leq i \leq n-1\}$. The vertex and edges set of $C(P_n)$ are defined by $V[C(P_n)] = \{vi : 1 \leq i \leq n\} \cup \{v_i' : 1 \leq i \leq n-1\}$ and $E[C(P_n)] = \{v_i v_i' : 1 \leq i \leq n-1\} \cup \{v_i' v_j' : 1 \leq i \leq n-2, i+2 \leq j \leq n\} \cup \{v_i' v_{i+1} : 1 \leq i \leq n-1\}$. Thus, S is the set of all vertices of P_n is the minimum geodetic set. By allotting the proper color, $\chi[C(P_n)]$ = 3. Thus, the vertex of S belongs to the different color class. Therefore, Sc is both geodetic and chromatic set of $C(P_n)$. Thus, $\chi_{gc}[C(P_n)]$ = n.

Remark: For the above theorem, we result that the $\chi_{gc}[M(P_n)] = \chi_{gc}[C(P_n)]$ = n.

Theorem 3.3: For any path P_n , n > 2. Then $\chi_{gc}[T(P_n)]$ = 4.

Proof: Consider the path P_n has vertex set $V(P_n) = \{v_i : 1 \leq i \leq n\}$ and the edge set $E(P_n)$ $\{e_i = v_i v_{i+1} : 1 \leq i \leq n-1\}$. By the concept of total graph of the path P_n, each edge $E(P_n)$ of P_n is subdivided by the vertices as $\{v_i' : 1 \leq i \leq n-1\}$. The vertex and edge set of $T(P_n)$ is given by $V[T(P_n)] = \{vi : 1 \leq i \leq n\} \cup \{v_i' : 1 \leq i \leq n-1\}$ where $\{v_i' : 1 \leq i \leq n-1\}$ is the vertices of $T(P_n)$ corresponding to the edge $\{v_i v_{i+1} : 1 \leq i \leq n-1\}$ of P_n and $E[T(P_n)] = \{vi\, v_{i+1} : 1 \leq i \leq n-1\} \cup \{v_i' v_{i+1}' : 1 \leq i \leq n-1\} \cup \{v_i v_i' : 1 \leq i \leq n\} \cup \{v_i' v_{i+1} : 1 \leq i \leq n-1\}$. Here $S = \{v_1, v_2'\ v_n\}$ be the geodetic set which is minimum. By allotting proper color, where as S_c is geodetic but not a chromatic set. Assume color C_1 to vertex v_1. Next assign color C_4 to the vertex v_2', color C_3 to the vertex v_n and color C_2 to the vertex v_2 and assign the same color class to remaining vertex without loss of generality. $\chi[T(P_n)]$ = 4. Therefore, S receives the different colors but not chromatic. By taking another vertex from $T(P_n)$ which belongs to the various color classes. That is, if the neighbor $N(v_1) \subseteq S$ has vertex with distinct colors. Thus, $S_c = S \cup N(v_1)$ is a geo chromatic number of $T(P_n)$ which is minimum. Hence, $\chi_{gc}[T(P_n)]$ = 4.

4.MIDDLE, CENTRAL AND TOTAL GRAPH OF CYCLE

Theorem 4.1: For any Cycle C_n , n >3. Then $\chi_{gc}[M(C_n)] = \begin{cases} 5, & n = 4 \\ n, & n > 4 \end{cases}$.

Proof: Consider C_n be a cycle of order n with the vertices $v_1, v_2, v_3, \ldots, v_n$. By the concept of middle graph, each edge of the cycle C_n is subdivided by the vertex v_m' where m = 1, 2, 3, …, n. The vertex and edge set of $V(C_n) = \{v_i : 1 \leq i \leq n\}$ and $E(C_n)$ $\{e_i = v_i v_{i+1} : 1 \leq i \leq n-1\} \cup \{e_n = v_1 v_n\}$. Thus vertex set $V[M(C_n)] = \{v_i : 1 \leq i \leq n\} \cup \{v_i' : 1 \leq i \leq n\}$ and edge set $E[M(C_n)] = \{v_i' v_{i+1} : 1 \leq i \leq n-1\} \cup \{v_i' v_{i+1}' : 1 \leq i \leq n-1\} \cup \{v_i v_i' : 1 \leq i \leq n\} \cup \{v_n' v_1\} \cup \{v_n' v_1'\}$. Hence, the set of all vertices of C_n is the minimum geodetic of $M(C_n)$.

Case (i): When n = 4, consider $\{v_1, v_2, v_3, v_4\}$ be the vertex set of cycle C_4. By the definition of middle graph each edge of the cycle C_4 is subdivided by a vertex $\{v_1', v_2', v_3', v_4'\}$ is $M(C_4)$. Here, $S = \{v_1, v_2, v_3, v_4\}$ be the minimum geodetic. Thus by proper coloring of $M(C_4)$, $\chi[M(C_n)]$ = 3. Let us allot color 'a' to

v_1, v_2 and v_3', color 'b' to v_1', v_3 and v_4 and color 'c' to v_2' and v_4'. Therefore, S is not a chromatic set, such that $\chi_{gc}[M(C_n)] > 4$. Choose the neighbor of $N(v_1) \subseteq S$ has a vertices of distinct colors, that is v_1' and v_4'. Hence, the color class of v_1' is already contained in S. Therefore, $S_c = \{S\} \cup \{v_4'\}$ is a geo chromatic number of M(C4). Therefore, it result $\chi_{gc}[M(C_n)] = 5$.

Case (ii): When n > 4, Here S is the set of vertices of C_n which is the minimum geodetic set of $M(C_n)$. By allotting the proper coloring of $M(C_n)$, $\chi[M(C_n)] = 3$, whereas v_1 receives color 'a', v_3 receives color 'b' and v_4 receives color 'c' there by allotting the colors to remaining vertices respectively. Hence, S_c is both geodetic and chromatic set of $M(C_n)$ which is minimum. Clearly it follows that $\chi_{gc}[M(C_n)] = n$.

Theorem 4.2: For n >3, $\chi_{gc}[C(C_n)] = n.$

Proof: Consider the cycle C_n, n > 3 has the vertex set $\{v_i : 1 \leq i \leq n\}$ and edge set $\{v_i v_{i+1} : 1 \leq i \leq n-1\} \cup \{v_1 v_n\}$. By the concept of the central graph, each edge of the cycle C_n is subdivided by the new vertices as $\{v_i' : 1 \leq i \leq n\}$. The vertex and edge set of $C(C_n)$ are defined as $V[C(C_n)] = \{v_i : 1 \leq i \leq n\} \cup \{v_i' : 1 \leq i \leq n\}$ and $E[C(C_n)] = \{v_i v_i' : 1 \leq i \leq n-1\} \cup \{v_i' v_{i+1} : 1 \leq i \leq n-1\} \cup \{v_n' v_1\} \cup \{v_i v_j : 1 \leq i \leq n-2, i+2 \leq j \leq n\}$. Here, set of all vertices of C_n is the minimum geodetic set of $C(C_n)$. By allotting proper coloring to $C(C_n)$, Let us assign color 'a' to the vertex v_1 and v_2, color 'c' to the vertex v_3, color b to vertex v_4 and color the remaining vertex by using the same color class respectively, whereas $\chi[M(C_n)] = 3$. Thus $S_c = |S|$ is the minimum geo chromatic number of C(Cn). Hence $\chi_{gc}[C(C_n)] = n.$

Theorem 4.3: For n >2, $\chi_{gc}[T(C_n)] = \begin{cases} 4, n \text{ is odd} \\ 5, n \text{ is even} \end{cases}.$

Proof: Consider the cycle C_n, n > 3 has the vertex set $\{v_i : 1 \leq i \leq n\}$ and edge set $\{v_i v_{i+1} : 1 \leq i \leq n-1\} \cup \{v_1 v_n\}$. By the concept of the total graph, each edge of the cycle C_n is subdivided by the new vertices as $\{v_i' : 1 \leq i \leq n\}$. The vertex and edge set of $T(C_n)$ are defined as $V[T(C_n)] = \{vi : 1 \leq i \leq n\} \cup \{v_i' : 1 \leq i \leq n\}$ and $E[T(C_n)] = \{v_i v_i' : 1 \leq i \leq n\} \cup \{v_i v_{i+1} : 1 \leq i \leq n-1\} \cup \{v_i' v_{i+1}' : 1 \leq i \leq n-1\} \cup \{v_i' v_{i+1} : 1 \leq i \leq n-1\} \cup \{v_1' v_n'\} \cup \{v_1 v_n'\} \cup \{v_1 v_n\}$. Since the set $S = \left\{v_1, v'_{\frac{n+1}{2}}\right\}$ or $S = \left\{v_1, v_{\frac{n}{2}+1}, v_2', v_{n-1}'\right\}$ is the minimum geodetic set of G. Here we consider the following.

Case (i): When n is odd, $S = \left\{v_1, v'_{\frac{n+1}{2}}\right\}$ is a geodetic set of $T(C_n)$. By allotting the proper color of $T(C_n)$, the vertices in S belongs to different color class. Since, $S_c > |S| = 2$ is not a geo chromatic set. Thus $\chi[T(C_n)] = 4$. By adding a different color class to geodetic set S. Let $N(v_1)$ and $N\left(v'_{\frac{n+1}{2}}\right) \subseteq S$ has vertices with different color. Then $S_c = S \cup \{N(v_1)\} \cup \left\{N\left(v'_{\frac{n+1}{2}}\right)\right\}$ is a geo chromatic set of $T(C_n)$ but which is not minimum, that is $\chi[T(C_n)] > 4$. Hence, we obtain a minimum geo chromatic set by adding a vertex which is different color class other than contained in S. Hence, it is clear that $\chi_{gc}[T(C_n)] = 4$.

Case (ii): When n is even, S = $\left\{v_1, v_{\frac{n}{2}+1}, v'_2, v'_{n-1}\right\}$ is a minimum geodetic set of $T(C_n)$. By allotting the proper color, the vertices in S belongs to different color class but which is not chromatic set. Choose another vertex from $T(C_n)$ which belongs to different color class. Let $v_i \in c_j$, $j \neq 1,2,3$. If $v_i \subseteq S$, then $S_c = S \cup \{v_i\}$ is geodetic as well as chromatic set of $T(C_n)$. Hence, $\chi_{gc}[T(C_n)] = 5$.

5.Conclusion

We compute the geo chromatic number of Middle, Central and Total graph for path graph and cycle graph. The concept of geo chromatic can be extended for some other parameter of the graphs.

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