(gp)*- Closed Sets In Topological Spaces

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ABSTRACT: In this paper we introduce and study a new class of generalized closed sets called (gp)*-closed sets in topological spaces and investigate some of the basic properties. We analyse the relation between (gp)*-closed sets with already existing closed sets.

KEYWORDS: $(gp)^*$ closed sets, gp-closed sets, g^*p -closed sets and g^* closed sets.

I INTRODUCTION

Levine[7] introduced generalized closed sets (briefly g-closed sets) in topological spaces and studied their basic properties. Mahi.H [12] introduced and studied gp -closed sets. Veerakumar [10] introduced g*-closed sets in topological spaces and studied their properties. Veerakumar [13] introduced and studied g*p -closed sets in topological spaces. The aim of this paper is to introduce a new class of generalized closed sets called (gp)*-closed sets and (gp)*- open sets.

II PRELIMINARIES

Definition 2.1 A subset A of a topological space (X,τ) is called

(i) a semi-open set if A ⊆ cl(int(A)) and a semi-closed set if int(cl(A)) ⊆A,
(ii) a preopen set if A ⊆ int(cl(A)) and a preclosed set if cl(int(A)) ⊆ A,
(iii) an α- open set if A ⊆ int(cl(int(A))) and an α -closed set if cl(int(cl(A))) ⊆ A,
(iv) a semi-preopen set if A⊆ cl(int(cl(A))) and a semi-preclosed set if int(cl(int(A))) ⊆A
(v) a regular open set if A=int(cl(A))and a regular closed set if cl(int(A))=A.

The semi-closure (resp.preclosure , semi-preclosure) of a subset A of a space (X,τ) is the intersection of all semi-closed(resp. preclosed , α-closed, semi-preclosed) sets that contain A and is denoted by scl(A) (resp.pcl(A), Acl(A), spcl(A)).

Definition 2.2 A subset A of a space (X,τ) is called

(i) a generalized closed (briefly g-closed) set[10] if cl(A)⊆U whenever A⊆U and U is open in (X,τ); the compliment of a g-closed set is called a g-open set,
(ii) a semi-generalized closed (briefly sg-closed) set[2] if scl(A)⊆U whenever A⊆U and U is semi-open in(X,τ); the compliment of sg-closed set is called a sg-open set,

(iii) a generalized semi-closed (briefly gs-closed) set if scl(A)⊆U whenever A⊆U and U is open in (X,τ)
(iv) an α-generalized closed (briefly αg-closed) set[3] if αcl(A)⊆U whenever A⊆U and U is α-open in (X,τ),
(v) a generalized α-closed (briefly gα-closed) set [3] if αcl(A)⊆U whenever A⊆U and U is α-open in (X,τ)
(vi) a g*- closed set [10] if cl(A)⊆U whenever A⊆U and U is g-open in (X,τ),
(vii) a g^{**}-closed set [8] if cl(A)⊆U whenever A⊆U and U is g*-open in (X,τ),
(viii) a generalized preclosed(briefly gp-closed) setif pcl(A)⊆U whenever A⊆U and U is open in (X,τ),
(ix) a generalized semi-preclosed (briefly gsp-closed) set [5] if spcl(A)⊆U whenever A⊆U and U is open in(X,τ)
(x) a generalized pre regular closed (briefly gpr-closed) set [6] if pcl(A)⊆U whenever A⊆U and U is regular open in (X,τ),
(xi) a $g^{\#}$-closed set [11] if cl(A)⊆U whenever A⊆U and U is αg-open in (X,τ),
(xii) a generalized α**-closed (briefly gα**-closed) set [3] if αcl(A)⊆int(cl(U)) whenever A⊆U and U is α-open in (X,τ),
(xiii) a g*s- closed set [9] if scl(A)⊆U whenever A⊆U and U is gs-open in (X,τ).
(xiv) A generalized pre closed set [12] if pcl (A))⊆U whenever A⊆U and U is open in (X,τ).
(xv) A g^{*}-preclosed set [13] if pcl (A))⊆U whenever A⊆U and U is g-open in (X,τ).

The compliment of the above mentioned sets are called their respective open sets.

III (gp)* - CLOSED SETS

Definition 3.1 A subset A of a topological space (X, τ) is said to (gp)*--closed set if scl(A) ⊆U whenever A⊆U and U is gp -open in (X,τ).
The class of all (gp) *-closed sets of (X, τ) is denoted by $(gp)^{*}$ C(X,τ).
Theorem 3.2 Every closed set in (X, τ) is (gp) *--closed .
Proof follows from the definition.
The following example supports that $(gp)^{*}$ closed set need not be true in general.
Example 3.3 Let X = {a,b,c} with topology τ = {φ,X,{a},{c },{a,c}}. Then the subset {a } is (gp) *-closed but not closed in (X,τ).
Theorem 3.4 Every g^{*} closed set in (X, τ) is (gp)*-closed .
Proof: Let A be g^{*} closed. Let A ⊆ U and U be g – open . Since A is g^{*} closed , then scl (A) ⊆ cl (A) ⊆ U. Hence A is (gp)*-closed .
The converse of the above theorem need not be true by the following example.
Example 3.5 Let X = {a,b,c} with topology τ = {φ,X,{a},{c), {a,c}}. Then the subset {a}, { c } are (gp) *-closed but not g^{*} closed in (X,τ).
Theorem 3.6 Every (pg)- closed set in (X, τ) is (gp)*-closed .
Proof: Let A be pg closed. Let A ⊆ U and U be preopen . Since A is pg closed , then scl (A) ⊆ pcl (A) ⊆ U. Hence A is (gp)*-closed.
The converse of the above theorem need not be true by the following example.
Example 3.7 Let X = {a,b,c} with topology τ = {φ,X,{a},{b), {a,b}}. Then the subset {a}, { b } are (gp) *-closed but not pg closed in (X,τ).

Theorem 3.8 : Every $g^{*}p$ - closed set in (X, τ) is (gp) *-closed .
Proof: Let A be $g^{*}p$ closed. Let A ⊆ U and U be g - open . Since A is $g^{*}p$ closed , then scl (A) ⊆ pcl (A) ⊆ U. Hence A is (gp)*-closed.
The converse of the above theorem need not be true by the following example.
Example 3.9 Let X = {a,b,c} with topology τ = {φ,X,{a},{b},{a,b}}. Then the subset {a} is (gp)*-closed but not $g^{*}p$ - closed in (X,τ).
Theorem 3.10: Every gp*- closed set in (X, τ) is (gp)*-closed .
Proof: Let A be gp* closed. Let A ⊆ U and U be gp - open . Since A is gp* closed , then scl (A) ⊆ cl (A) ⊆ U. Hence A is (gp)*-closed
The converse of the above theorem need not be true by the following example.
Example 3.11 Let X = {a,b,c} with topology τ = {φ,X,{a},{b},{a,b}}. Then the subset {a}, {a,b} are (gp)*-closed but not gp*- closed in (X,τ).
Theorem 3.12 :Every g# - closed set in (X, τ) is (gp)*-closed.
Proof: Let A be g# closed. Let A ⊆ U and U be αg - open . Since A is g# closed , then scl (A) ⊆ cl (A) ⊆ U. Hence A is (gp)*-closed.
The converse of the above theorem need not be true by the following example.
Example 3.13: Let X = {a,b,c} with topology τ = {φ,X,{a},{b},{a,b}}. Then the subset {b}, {a,b} are (gp)*-closed but not g#- closed in (X,τ).
Theorem 3.14 :Every gs - closed set in (X, τ) is (gp)*-closed.
Proof: Let A be gs closed. Let A ⊆ U and U be open . Since A is gs closed , then scl (A) ⊆ cl (A) ⊆ U. Hence A is (gp)*-closed.
The converse of the above theorem need not be true by the following example.
Example 3.15: Let X = {a,b,c} with topology τ = {φ,X,{a},{b},{a,b}}. Then the subset {a,b} is (gp)*-closed but not gs- closed in (X,τ).
Theorem 3.16 :Every αg - closed set in (X, τ) is (gp)*-closed.
Proof: Let A be $\alpha g -$ closed. Let A ⊆ U and U be open . Since A is $\alpha g -$ closed , then scl (A) ⊆ αcl (A) ⊆ U. Hence A is (gp)*-closed.
The converse of the above theorem need not be true by the following example.
Example 3.17: Let X = {a,b,c} with topology τ = {φ,X,{a},{b},{a,b}}. Then the subset {a,b} , {a} are (gp)*-closed but not αg - closed in (X,τ).
Theorem 3.18 :Every αg^{*} - closed set in (X, τ) is (gp)*-closed.
Proof: Let A be $\alpha g^{*} -$ closed. Let A ⊆ U and U be αg - open . Since A is $\alpha g^{*} -$ closed , then scl (A) ⊆ αcl (A) ⊆ U. Hence A is (gp)*-closed.
The converse of the above theorem need not be true by the following example.
Example 3.19: Let X = {a,b,c} with topology τ = {φ,X,{a},{b},{a,b}}. Then the subset {a,b} , {a} are (gp)*-closed but not αg^{*} - closed in (X,τ).
Theorem 3.20 :Every g^{**} - closed set in (X, τ) is (gp)*-closed.
Proof: Let A be $g^{**} -$ closed . Let A ⊆ U and U be g^{*} - open . Since A is $g^{**} -$ closed , then scl (A) ⊆ cl (A) ⊆ U. Hence A is (gp)*-closed.
The converse of the above theorem need not be true by the following example.
Example 3.21: Let X = {a,b,c} with topology τ = {φ,X,{a},{b},{a,b}}. Then the subset {a} is (gp)*-closed but not g^{**} - closed in (X,τ).
Theorem 3.22 : A is a $(gp)^{*}$-closed set of (X, τ) then $scl(A)\backslash A$ does not contain any nonempty generalized-preclosed set.
Proof : Let F be a generalized -preclosed set of (X, τ) such that F ⊆ $scl(A)\backslash A$. A ⊆ X\F . A is $(gp)^{*}$-closed and X\F is generalized-preopen such that A ⊆ X\F. Therefore

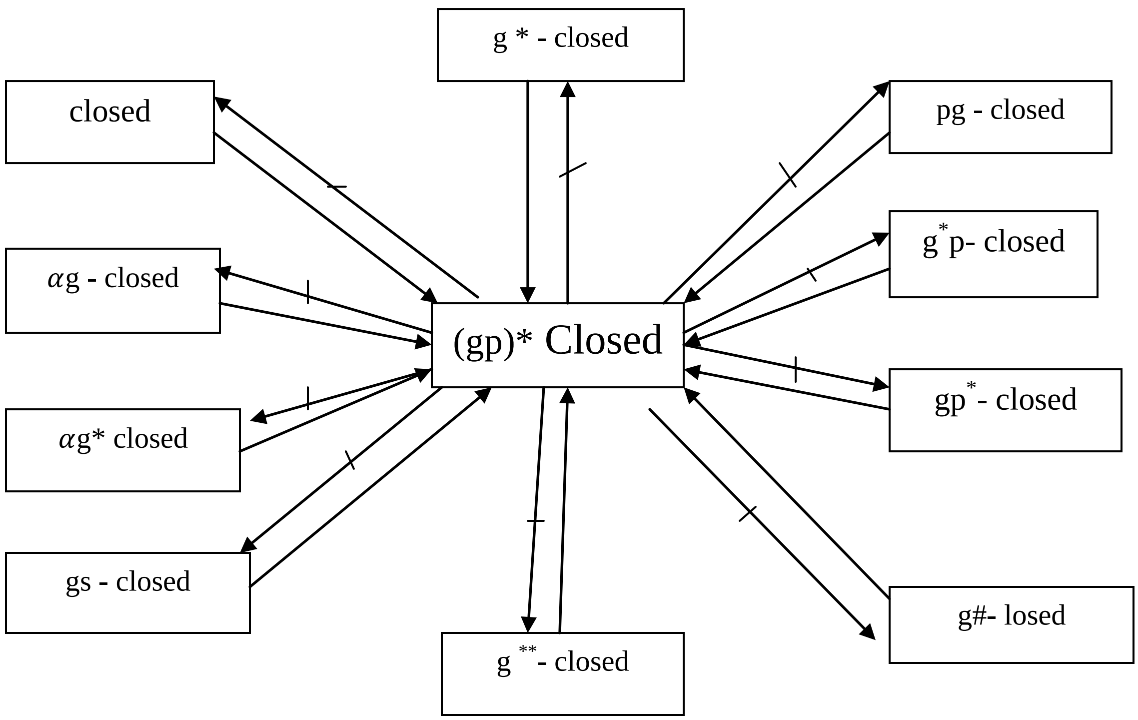
$scl(A) \subseteq X \backslash F$.implies $F \subseteq X \backslash scl(A)$. So, $F \subseteq ((X \backslash scl(A)) \cap scl(A) \backslash A)$,Therefore $F = \phi$

Theorem 3.25 : If A is a $(gp)^*$-closed set of (X, τ) such that $A \subseteq B \subseteq scl(A)$, then B is a $(gp)^*$-closed set of (X, τ).

Proof : Let U be a generalized-preopen set such that $B \subseteq U$, then $A \subseteq B \subseteq U$. We have to prove that $scl(B) \subseteq U$. $scl(A) \subseteq U$ since A is a $(gp)^*$-closed.$B \subseteq scl(A)$. implies $scl(B) \subseteq scl(scl(A)) = scl(A) \subseteq U$ .implies $scl(B) \subseteq U$.therefore B is a $(gp)^*$-closed set.

Remark3.26 : The following diagram shows the relationships establish between

(gp)* - closed sets and some other sets. A $\rightarrow$ B represent A implies B but B need not imply A.



IV (gp)* - OPEN SETS

Definition 4.1 A subset A of a topological space (X, τ) is said to (gp)*--open set if A^c is $(gp)^*$ - closed .

Theorem 4.2 : If A and B are $(gp)^*$ - opensets, then $A \cap B$ is also $(gp)^*$ - open.

Proof: A and B are $(gp)^*$ - opensets.Therefore A^c and B^c are $(gp)^*$ - closed sets.therefore $A^c \cup B^c$ are $(gp)^*$ - closed.$A^c \cup B^c = (A \cap B)^c$ is $(gp)^*$ - closed. Therefore $A \cap B$ is $(gp)^*$ - open.

Theorem 4. 3: A subset A of (X, τ) is $(gp)^*$ - open if and only if $F \subseteq int(A)$ whenever F is gpclosed

Proof: Suppose that F is generalized pre closed.$F \subseteq A$ then $F \subseteq int(A)$.Then we have to prove that A is$(gp)^*$open.Let G be generalized preopen and $A^c \subseteq G$.Then $G^c \subseteq A$ and G^c is generalized preclosed.Thus $G^c \subseteq$ int(A) .it follows that int(A^c)$\subseteq$G.Hence A^c is $(gp)^*$-closed and hence A is $(gp)^*$-open.Conversely, suppose that A is $(gp)^*$-open $F \subseteq A$ and F is generalized preclosed. F^c is generalized pre closed and $A^c \subseteq F^c$.cl(A^c) $\subseteq F^c$ and hence $(int(A))^c \subseteq F^c$.Thus $F \subseteq$ int(A^c).

Theorem 4. 4: If $A \subseteq M \subseteq X$ where A is $(gp)^*$ open relative to M and M is $(gp)^*$ open in X, then A is $(gp)^*$ open in X.

Proof: Let F be a generalized pre-closed set in X and suppose that $F \subseteq A$.Then there exists a closed set K such that int(K)$\subseteq F \subseteq$K.therefore int(K)$\cap M \subseteq F \subseteq K \cap M$. since (int(K))$\cap M \supseteq$ /M-int($K \cap M$) and $K \cap M$ is M-closed, we have the set F is generalized pre-closed in M.But A is $(gp)^*$ open relative to M.therfore$F \subseteq$M-int(A).since M-int(A) is an openset relative to M, we have $F \subseteq G \cap M \subseteq A$, for some openset G in X.sinceM is $(gp)^*$ open inX, we have $F \subseteq$ int(M)$\subseteq$M.therfore $F \subseteq$(int(M))$\cap G \subseteq M \cap G \subseteq$A.It follows then that $F \subseteq$int(A).Hence A is $(gp)^*$ open in X.

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