ON FLEXURAL ENERGY DISSIPATION CHARACTERIZATION OF FRP LAMINATES USING 2D-ANALYTICAL MODELS

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Abstract: *The fiber reinforced composite materials are ideal for structural applications wherever high strength-to-weight and stiffness-to-weight ratios are needed. Composite materials are often tailored to fulfill the actual needs of stiffness and strength by changing lay-up and fiber orientations. The ability to tailor a material to its job is one among the most vital advantages of a composite material over a normal material. Therefore the research and development of composite materials within the design of aerospace, civil and mechanical structures has grown tremendously within the past few decades. In this paper the flexural damping behaviors of composites were characterized analytically during this study. Damping of laminates is calculated analytically by strain energy weighted dissipation method. The specific damping capacity (SDC) of the composite was determined in accordance to the energy dissipation concept, which was defined as the ratio of the dissipated energy to the stored energy for per circle of vibration. A 2-D analytical model was developed basing on Adams, Ni RG models. The 2-D analytical model was validated by the scrutiny of SDC of [0/60/60]s and [0/90/45/45]s laminates with the experimental information. A MATLAB code is developed for repetitive nature of calculations. SDC for unidirectional and cross ply laminates are predicted with Code developed.*

Keywords: **Flexural Vibration, Composite Laminates, Specific Damping Capacity, 2D- Models**

1. INTRODUCTION

Many researchers had carried out the work of enhancing the material damping in composites. Damping is an important feature of the dynamic behavior of the fiber reinforced composite structures involving minimization of resonant and near resonant vibrations. At the constituent level, the energy dissipation in fiber reinforced composites is induced by different mechanisms such as the visco-elastic nature of the matrix and fiber materials, the damping of the fiber matrix interphase, the damping due to damage etc. At the laminate level, the damping is strongly dependent on the constituent layer properties in addition layer orientations, inter-laminar effects, vibration coupling etc.

The damping of laminated composites has been developed by several investigators is inconclusive. Schultz and Tsai tested beams of different lamination geometry in bending but there were so many differences between their theoretical and experimental values. Schultz and Tsai considered theoretically and experimentally the complex dynamic moduli of (0°, -60°, 60)s and (0°, 90 °, -45 °, 45°)s, laminates in glass/epoxy but the agreement was not good. Clary investigated the effect of fiber orientation on the flexural vibration of plates and beams and measured the damping capacity. He did not find any apparent relationship between damping and fiber orientation but he said that the damping values were small. Adams and Bacon investigated the effect of laminate geometry and fiber orientation on the dynamic properties of unidirectional off-axis, angle-plied, cross-plied and generally-plied such as (0 °, -60 °, 60 °)sand (0 °, 90 °, -45 °, -45 °)s' laminates in both GFRP and CFRP. Adams and Maheri measured the specific damping capacity of two laminate configurations of glass fiber composites. At first it was a balanced symmetric angle-ply laminate, [-θ /+ θ/- θ /+θ]sand the second was a symmetric unidirectional laminate, [θ]their criterion was later used by Adams and Maheri together with the basic plane stress relations, top predict the moduli and the specific damping capacity of the anisotropic beams with respect to fiber orientation.

In this study, a 2-D analytical model was developed considering for all in-plane stress and strain quantities into the analysis of energy dissipation. Throughout the derivation, the distinct variations between alternative analytical models were indicated perspicuously.

2. THEORY OF ANALYTICAL MODELS

All theories compared during this work are supported the assumptions of classical laminate theory for orthotropic materials. In short, this suggests good bonding between plies, no out of plane stresses or strains and little, elastic deformations. Matrices of applied forces and moments are written as [N] and [M], severally, with subscripts accustomed indicate specific elements. The standard notation utilized in this paper for subscripts, unless otherwise noted, is range subscripts, i.e. 1, 2, or 12, are accustomed denote principal material directions of a plate whereas letter subscripts, i.e. x, y, or xy, denote the global coordinates of the laminate. The two coordinate systems are usually offset by associate degree angle, h. For elastic deformations, the strain energy keep in every ply is separated into three components: strain energy in the fiber direction, strain energy transverse to the fiber direction, and strain energy in shear. Some quantity of every of those elements is dissipated throughout deformation. One way to quantify this dissipation is by using the specific damping capacity that is outlined because the strain energy dissipated throughout a stress cycle, divided by the maximum strain energy throughout a stress cycle.

2.1 Adams and Bacon Theory

Adams and Bacon [3] presented a general equation to predict the SDC of beams for applied forces, {N}, and moments, {M}. The equation can be used for any laminate configuration and the lamina damping coefficients

$$\Psi_{laminate} = \frac{2\sum_{k=1}^{n} \int_{h_k}^{h_{k-1}} \delta(\Delta u_1) + \delta(\Delta u_2) + \delta(\Delta u_{12}) dz}{\{N\}^T \{\xi^o\} + \{M\}^T \{k\}} \tag{1}$$

2.2 Ni and Adam Theory

Additional damping prediction equations were developed by Ni and Adams [1] they assumed free flexure deformation, stress-independent damping coefficients and a symmetric layup. This final assumption leads to no mid-plane strains under classical laminate plate theory. In addition to neglecting σy and σxy, Ni and Adams also argued that the transverse strain

$$\begin{aligned}
\Psi_1 &= \frac{8\Psi_L}{C_{11}^* N^3} \sum_{K=1}^{N/2} (m^2 C_{11}^* + mnC_{16}^*)\left[m^2(\overline{Q_{11}^k} C_{11}^* + \overline{Q_{12}^k} C_{12}^* + \overline{Q_{16}^k} C_{16}^*)\right] W_K \\
\Psi_2 &= \frac{8\Psi_T}{C_{11}^* N^3} \sum_{K=1}^{N/2} (m^2 C_{11}^* - mnC_{16}^*)\left[n^2(\overline{Q_{11}^k} C_{11}^* + \overline{Q_{12}^k} C_{12}^* + \overline{Q_{16}^k} C_{16}^*))\right] W_K \\
\Psi_{12} &= \frac{8\Psi_{LT}}{C_{11}^* N^3} \sum_{K=1}^{N/2} (2mnC_{11}^* - (m^2 - n^2) C_{16}^*)\left[mn(\overline{Q_{11}^k} C_{11}^* + \overline{Q_{12}^k} C_{12}^* + \overline{Q_{16}^k} C_{16}^*)\right] W_K
\end{aligned} \tag{2}$$

e_y, in each lamina will be much smaller than the longitudinal and shear strains and could be neglected results equations.

Where [C] is the normalized flexural compliance of the laminate, [Qk] is the stiffness of the kth lamina, p is the total number of plies present in the laminate and Wk a weighting factor based on the position of the kth lamina within the laminate

2.3 Adams and Maheri Theory

Revisions to the Ni and Adams equations were made by Adams and Maheri [3]. The derivation of the equation deviated from the method used by Ni and Adams but the final result is similar. In short, the Adams and Maheri theory is the Ni and Adams theory with slight notational changes and the transverse lamina strain, ey term included

$$\Psi_1 = \frac{8\Psi_L}{C_{11}^* N^3} \sum_{K=1}^{N/2} (m^2 C_{11}^* + n^2 C_{12}^* + mnC_{16}^*) \left[m^2 (\overline{Q_{11}^k} C_{11}^* + \overline{Q_{12}^k} C_{12}^* + \overline{Q_{16}^k} C_{16}^*) \right] W_K$$
$$\Psi_2 = \frac{8\Psi_T}{C_{11}^* N^3} \sum_{K=1}^{N/2} (m^2 C_{11}^* + n^2 C_{12}^* - mnC_{16}^*) \left[n^2 (\overline{Q_{11}^k} C_{11}^* + \overline{Q_{12}^k} C_{12}^* + \overline{Q_{16}^k} C_{16}^*)) \right] W_K \quad (3)$$
$$\Psi_{12} = \frac{8\Psi_{LT}}{C_{11}^* N^3} \sum_{K=1}^{N/2} (2mnC_{11}^* - 2mnC_{12}^* - (m^2 - n^2) C_{16}^*) \left[mn (\overline{Q_{11}^k} C_{11}^* + \overline{Q_{12}^k} C_{12}^* + \overline{Q_{16}^k} C_{16}^*) \right] W_K$$

The contributions from all in-plane stresses but does not allow for out-of-plane contributions to energy dissipation. The latter limitation is consistent for all theories presented in this paper. The number of terms involved in Eqs (3) leads to a level of complexity that can typically only be handled using numerical techniques. Adams and Bacon recognized this and derived several approximations to Eq. that lend themselves to easier solutions by neglecting certain stresses and strains or limiting the allowable layups

2.3 Lin-Tsai Model

For a symmetric laminated composite subjected to pure bending moment Mx, the corresponding curvature is given by [14]

$$\begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix} = \begin{bmatrix} D_{11}^* & D_{12}^* & D_{16}^* \\ D_{21}^* & D_{22}^* & D_{26}^* \\ D_{16}^* & D_{26}^* & D_{66}^* \end{bmatrix} \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} \quad (4)$$

Apply Mx≠ 0, My = 0 & Mxy= 0

$$k_x = D_{11}^* M_x \qquad k_y = D_{12}^* M_x \qquad k_{xy} = D_{16}^* M_x \quad (5)$$

$$C_{ij}^* = \frac{1}{D_{ij}^*} = \frac{1}{D_{ij}} h^* = C_{ij} h^* \quad (6)$$

Where h*=h3/12,h is the thickness of the composite laminate. Substitute Eqn. (6) in Eqn.(5) then From the thin

$$k_x = \frac{C_{11}^*}{h^*} M_x \qquad k_y = \frac{C_{12}^*}{h^*} M_x \qquad k_{xy} = \frac{C_{16}^*}{h^*} M_x \quad (7)$$

plate assumption. The strain in a material is proportional to the distance measured from the mid plane of laminate in terms of the curvature. Combining Eqns. (6) and (7) leads to the relationship of the strain components to the mending moment Mx as

$$\varepsilon_x = z \frac{C_{11}^*}{h^*} M_x \qquad \varepsilon_y = z \frac{C_{12}^*}{h^*} M_x \qquad \varepsilon_{xy} = z \frac{C_{16}^*}{h^*} M_x \quad (8)$$

3. SPECIFIC DAMPING CAPACITY

From the energy dissipation concept the SDC of materials can be defined as the ratio of the dissipated energy to the stored energy for per circle of vibration

$$\Psi = \frac{\Delta U}{U} \tag{9}$$

Where ΔU = Dissipated energy per cycle of vibration and U= Total strain energy stored in a system when deformation is maximum

For the composite laminates under cyclic vibration, the energy dissipation can be evaluated from the summation of those calculated individually from the fiber direction, transverse direction, and in-plane shear direction as [2]

$$\Delta U = \Delta U_1 + \Delta U_2 + \Delta U_{12} \tag{10}$$

Combining Eqn. (9) and (10), we get SDC in terms of fibre, transverse and in-plane shear directions.

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$$\psi = \frac{\Delta U_1}{U} + \frac{\Delta U_2}{U} + \frac{\Delta U_{12}}{U} \tag{11}$$

$$\begin{aligned} \Delta U_1 &= \Psi_L U_1 \\ \Delta U_2 &= \Psi_T U_2 \\ \Delta U_{12} &= \Psi_{LT} U_{12} \end{aligned} \tag{12}$$

Where ΨL, ΨT, and ΨLT indicate the SDC of a unidirectional composite in the fiber, transverse and in-plane shear directions, respectively U1, U2, and U12 denote the corresponding strain energy in the fiber, transverse and in-plane shear directions as well

$$\Delta U_1 = \Psi_L U_1 = \Psi_L \frac{1}{2} \int_v \varepsilon_1 \sigma_1 dv \tag{13}$$

From the coordinate transformation relation, the stress and strain components in the fiber direction can be correlated to the corresponding components in the x–y coordinate. Substitute and from [14] in Eqn. (13)

$$\Delta U_1 = \frac{1}{2} \int_v \Psi_L (m^2 \varepsilon_x + n^2 \varepsilon_y + mn\varepsilon_{xy})(m^2 \sigma_x + n^2 \sigma_y + 2mn\sigma_{xy}) dv \tag{14}$$

Where m = cosθ and n=sinθ and 'θ' is the fibre orientation with respect to x-y coordinate. Substituting Eqn.(8) in the energy dissipation in Eqn.(14) can be written as

$$\begin{aligned} \Delta U_1 = \frac{1}{2} \int_v \Psi_L (m^2 C_{11}^* + n^2 C_{12}^* + mnC_{16}^*)(m^2 (\overline{Q_{11}^k} C_{11}^* + \overline{Q_{12}^k} C_{12}^* + \overline{Q_{16}^k} C_{16}^*) \\ + n^2 \overline{Q_{12}^k} C_{11}^* + \overline{Q_{22}^k} C_{12}^* + \overline{Q_{26}^k} C_{16}^*) + 2mn(\overline{Q_{16}^k} C_{11}^* + \overline{Q_{26}^k} C_{12}^* + \overline{Q_{66}^k} C_{16}^*) M_x \left(\frac{z}{h^*} \right)^2 dv \end{aligned} \tag{15}$$

Where Q is the Compliance matrix for the off axis composites. When the dimensions of the laminated beam are assumed to be L in length, h in thickness and unitin width, the above integration further expressed as

$$\begin{aligned} \Delta U_1 = 2 \int_0^{L/2} M_x^2 dx \int_0^{h/2} \Psi_L (m^2 (\overline{Q_{11}^k} C_{11}^* + \overline{Q_{12}^k} C_{12}^* + \overline{Q_{16}^k} C_{16}^*) + n^2 \overline{Q_{12}^k} C_{11}^* + \overline{Q_{22}^k} C_{12}^* + \overline{Q_{26}^k} C_{16}^*) + \\ 2mn(\overline{Q_{16}^k} C_{11}^* + \overline{Q_{26}^k} C_{12}^* + \overline{Q_{66}^k} C_{16}^*) \left(\frac{z}{h^*} \right)^2 dv \end{aligned} \tag{16}$$

Where Eqn. (16) indicates the total strain energy dissipation in the fibre direction. A laminated plate subjected to a Bending moment Mx; the total strain energy is written as

$$U = \frac{1}{2}\int_0^l M_x k_x dx \tag{17}$$

$$= \frac{C_{11}^*}{h^*}\int_0^l M_x^2 dx \tag{18}$$

From the energy dissipation concept; SDC of composite laminate in the fibre direction Ψ_1 can be written as

$$\psi_1 = \frac{2\int_0^{l/2} M_x^2 dx \int_0^{h/2} \Psi_L (m^2(\overline{Q_{11}^k}C_{11}^* + \overline{Q_{12}^k}C_{12}^* + \overline{Q_{16}^k}C_{16}^*) + n^2\overline{Q_{12}^k}C_{11}^* + \overline{Q_{22}^k}C_{12}^* + \overline{Q_{26}^k}C_{16}^*) + 2mn(\overline{Q_{16}^k}C_{11}^* + \overline{Q_{26}^k}C_{12}^* + \overline{Q_{66}^k}C_{16}^*)\left(\frac{z}{h^*}\right)^2 dv}{\frac{C_{11}^*}{h^*}\int_0^{l/2} M_x^2 dx} \tag{19}$$

$$\psi_1 = \frac{2}{C_{11}^* h^*}\int_0^{h/2} \Psi_L (m^2(\overline{Q_{11}^k}C_{11}^* + \overline{Q_{12}^k}C_{12}^* + \overline{Q_{16}^k}C_{16}^*) + n^2\overline{Q_{12}^k}C_{11}^* + \overline{Q_{22}^k}C_{12}^* + \overline{Q_{26}^k}C_{16}^*) + 2mn(\overline{Q_{16}^k}C_{11}^* + \overline{Q_{26}^k}C_{12}^* + \overline{Q_{66}^k}C_{16}^*) z^2 dz \tag{20}$$

in Eqn.(20) indicates the SDC of composite laminates in fibre direction .The integral operation in Eqn.(20) can be replaced by summation operator as

$$\Psi_1 = \frac{8\Psi_L}{C_{11}^* N^3}\sum_{K=1}^{N/2}(m^2C_{11}^* + n^2C_{12}^* + mnC_{16}^*)[(m^2(\overline{Q_{11}^k}C_{11}^* + \overline{Q_{12}^k}C_{12}^* + \overline{Q_{16}^k}C_{16}^*) + n^2\overline{Q_{12}^k}C_{11}^* + \overline{Q_{22}^k}C_{12}^* + 1\overline{Q_{26}^k}C_{16}^*) + 2mn(\overline{Q_{16}^k}C_{11}^* + \overline{Q_{26}^k}C_{12}^* + \overline{Q_{66}^k}C_{16}^*) \mathrm{W}_k \tag{21}$$

Where Wk = (k3- (k -1)3) is the weighting factor in the kth layer. In the same way, the damping capacity in the transverse and in-plane shear direction can also be calculated, respectively

$$\Psi_2 = \frac{8\Psi_T}{C_{11}^* N^3}\sum_{K=1}^{N/2}(n^2C_{11}^* + m^2C_{12}^* - mnC_{16}^*)[(n^2(\overline{Q_{11}^k}C_{11}^* + \overline{Q_{12}^k}C_{12}^* + \overline{Q_{16}^k}C_{16}^*) + m^2\overline{Q_{12}^k}C_{11}^* + \overline{Q_{22}^k}C_{12}^* + 1\overline{Q_{26}^k}C_{16}^*) - 2mn(\overline{Q_{16}^k}C_{11}^* + \overline{Q_{26}^k}C_{12}^* + \overline{Q_{66}^k}C_{16}^*) \mathrm{W}_k \tag{22}$$

$$\Psi_2 = \frac{8\Psi_T}{C_{11}^* N^3}\sum_{K=1}^{N/2}(n^2C_{11}^* + m^2C_{12}^* - mnC_{16}^*)[(n^2(\overline{Q_{11}^k}C_{11}^* + \overline{Q_{12}^k}C_{12}^* + \overline{Q_{16}^k}C_{16}^*) + m^2\overline{Q_{12}^k}C_{11}^* + \overline{Q_{22}^k}C_{12}^* + 1\overline{Q_{26}^k}C_{16}^*) - 2mn(\overline{Q_{16}^k}C_{11}^* + \overline{Q_{26}^k}C_{12}^* + \overline{Q_{66}^k}C_{16}^*) \mathrm{W}_k \tag{23}$$

Therefore total specific damping capacity in a composite laminate can be calculated by summing up all the specific damping capacity in fibre, transverse and in plane shear directions i.e. Substituting Eqns. (21), (22) and (23) in Eqn. (24), We get total damping capacity

$$\psi = \psi_1 + \psi_2 + \psi_{12} \tag{24}$$

Table1 .Material properties of carbon/epoxy and kevlar/epoxy composites

Materials	Effective Elastic Properties						
	E_L (GPA)	**E_T (GPA)**	**E_{LT} (GPA)**	**μ_{LT}**	**ψ_L**	**ψ_T**	**ψ_{LT}**
CARBON FIBER/DX210	172.7	7.20	3.76	0.29	0.4	4.2	7.03
KEVLAR FIBER/SR1500	50.7	4.50	2.10	0.33	9.4	15.	23.8

4. RESULTS AND DISCUSSIONS

The flexural damping responses of the Carbon epoxy and Kevlar epoxy composite laminates [0/-60/60]s and [0/90/45/-45] obtained from the analytical models are compared with the Ni Adams model and Adams–Maheri model in below figures

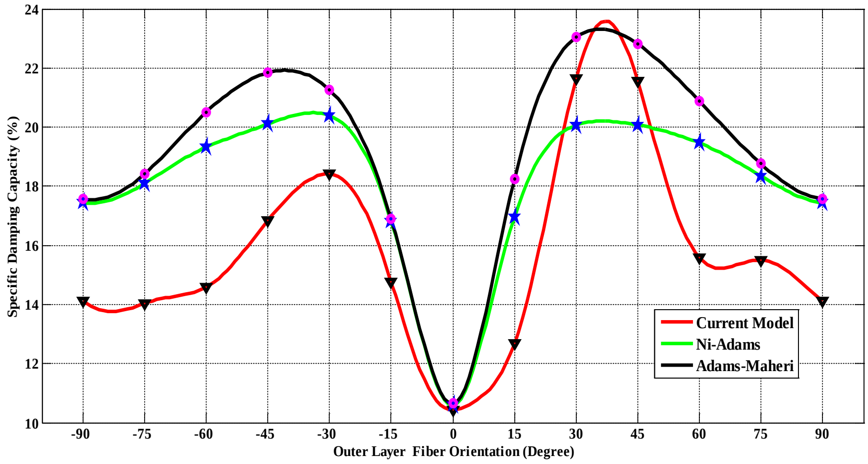

Fig1. Flexural damping properties of Carbon/epoxy laminates [0 -60 60 60 -60 0]

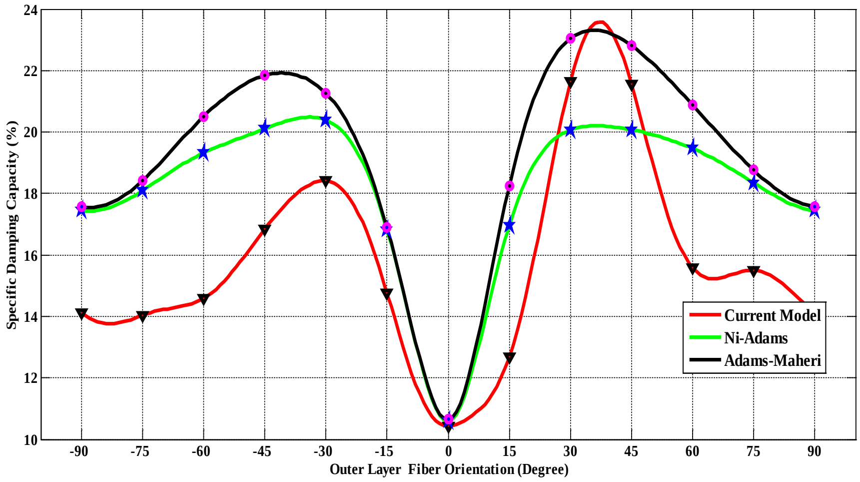

Fig.2 Flexural damping properties of Kevlar/epoxy laminates [0 -60 60 60 -60 0]

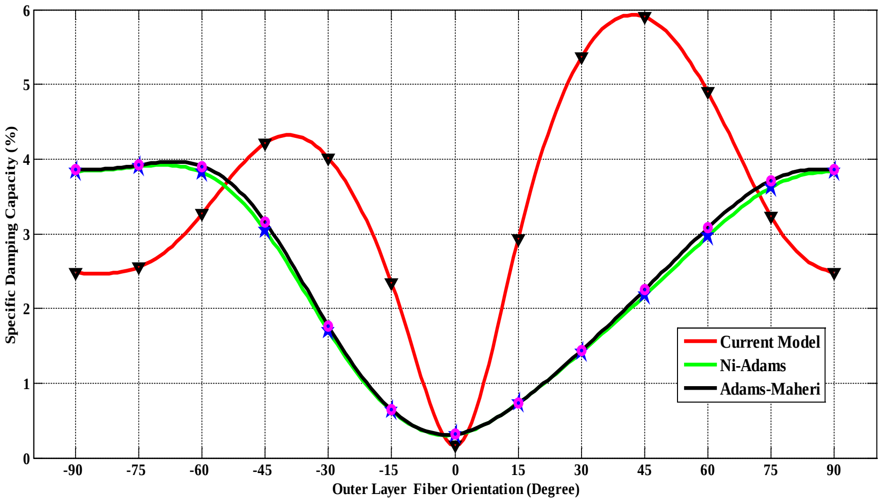

Fig.3 Flexural damping properties of Carbon/epoxy laminates [0 90 -45 45 45 -45 90 0]

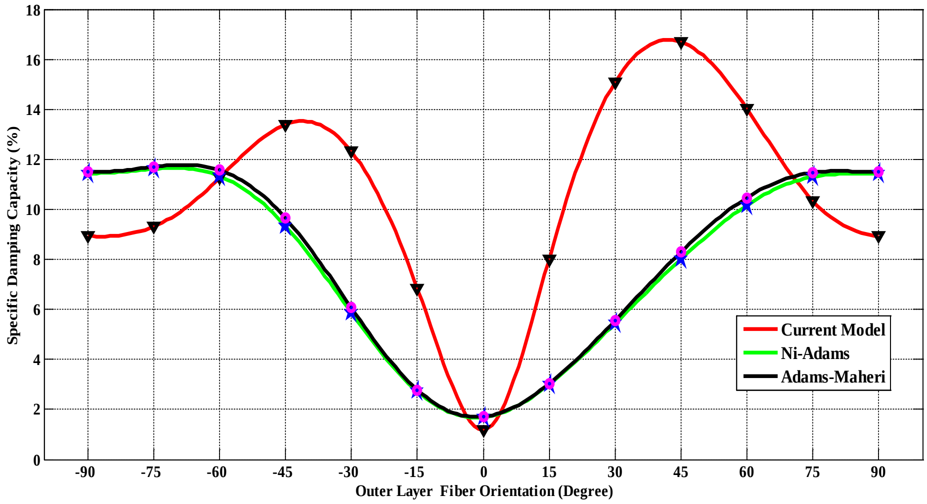

Fig.4 Flexural damping properties of Kevlar/epoxy laminates [0 90 -45 45 45 -45 90 0]

All calculations were performed using MATLAB Unfortunately, the general theory of Adams and Bacon can only be compared with the data presented in the original paper. This is because the theory allows for stress dependent damping coefficients but does not explicitly identify the relationship between the coefficients and the lamina stresses. The results of Adams and Bacon could not be recalculated for other laminate geometries. Transcription of data points from the original plots has been used to recreate their curves in the current work. Adams and Maheri measured the damping capacity of two laminate configurations of Fibredux 913/ glass fiber composites. The first was a balanced symmetric angle-ply laminate, [-θ/ +θ /-θ/ +θ] and the second was a symmetric unidirectional laminate, [θ]8. Beams were cut from plates with the above layups and excited at the fundamental flexural modes of vibration.

5. CONCLUSIONS

The 2-D models for characterizing flexural damping properties of composite laminates were reviewed in this study. It was found from that when the in-plane stress and strain quantities that were ignored in the Ni–Adams and Adams–Maheri models were included in the present model for the calculation of energy dissipation, the results were the same as those deduced from the Saravanos Chamis model. Compared to the Ni Adams and Adams Maheri models, the present models demonstrate good agreement with the experimental data. From the above results specific damping capacity is superior in Kevlar when compared to carbon

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