Analysis of Stability in SIQRA Model at Equilibrium Point

Dr. Kaveri Kanchan Kumari [1] , Dr. Ashish Kumar Jha[2]
[1]Research Scholar, University Department of Mathematics, Ranchi University, Ranchi
[2]Associate Professor, , University Department of Mathematics, Ranchi University, Ranchi

Abstract: *In this paper a mathematical model is proposed ,which is used to control the viruses in computer network. For this the proposed model is SIQRA (Susceptible-Infected-Quarantine-Recovered-Antidotal) model, which is extension of SAIR (Susceptible-Infected-Quarantine-Recovered) model. In this paper we discussed about Virus Free Equilibrium Point and Endemic Equilibrium point.*

Keywords: *Quarantine, Antidotal, Virus Free Equilibrium¸ Endemic Equilibrium.*

1. Introduction

In Ancient time the invention of hardware gave many changes in human life. Just like this when software technology developed, the human life became very easier. The computer became an essential tool for everyday life. But computer facing many challenges in the form of malicious objects. Computer viruses came in the form of programs. Nowaday malicious objects have become a major threat for cyber world.. They have power of stealing the personal data, fioles and other valuable informations. Malicious objects throughout a network can be studied by using epidemiological models for VFE. Various computer virus model are proposed [1-5]. The most common models are Susceptible- Infectious- Susceptible (SIS) model[6-7], Susceptible- Infected- Recovered (SIR) model[8-9], Susceptible- Exposed- Infected- Recovered (SEIR) model [10], Susceptible-Exposed- Infected-Antidotal-Recovered (SEIAR) model[11], Susceptible- Exposed- Infected- Quarantine- Recovered-Susceptible (SEIQS) model [12], Kaveri et.al.[14,15] discussed the different type of model.

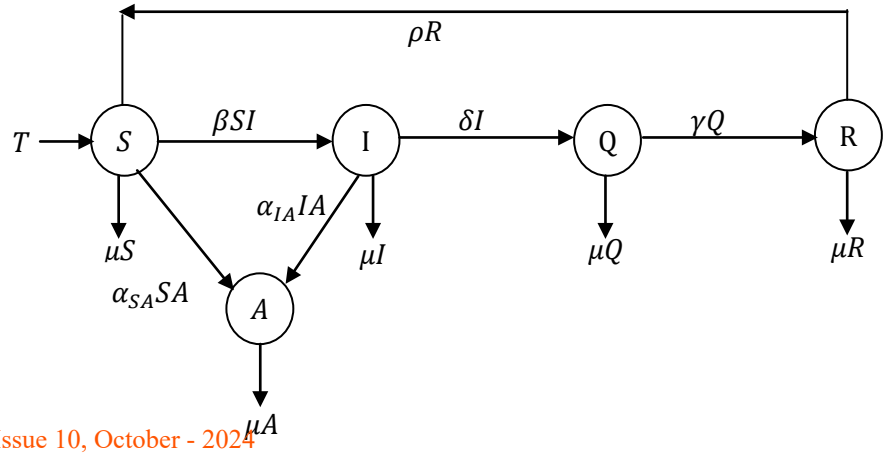
2. Mathematical Formulation of model

The model that is presented here is based on the SAIR model. The proposed model is SIQRA models, which is an extension of SAIR model. We introduce a new compartment Q, that is Quarantine compartment. In this model

$S(t)$: a susceptible compartment
$I(t)$: an infective compartment
$Q(t)$: a quarantine compartment
$R(t)$: a recovered or immune compartment
$A(t)$: a antidotal compartment

The total population N is divided into five compartment.

where T is influx rate, that is same new computers are added to the network K; β is infection rate of susceptible computers; μ is the mortality rate from all causes of death for a population; δ is quarantined rate of infected computers; γ is recovered rate of quarantined computers; ρ is recovering rate of removed computers, α_{SA} is conversion of susceptible computers into antidotal compartments, α_{IA} represents infected computers that can be fixed by using antivirus programs being converted into antidotal compartments.



Schematic diagram for malicious objecs in computer network

The SIQRA model for malicious object propagation was proposed and can be describe by

$$\left.\begin{aligned}\dot{S}(t) &= T - \alpha_{SA}SA - \mu S - \beta SI + \rho R\\ \dot{I}(t) &= \beta SI - \alpha_{IA}IA - \mu I - \delta I\\ \dot{Q}(t) &= \delta I - \mu Q - \gamma Q\\ \dot{R}(t) &= \gamma Q - \mu R - \rho R\\ \dot{A}(t) &= \alpha_{SA}SA + \alpha_{IA}IA - \mu A\end{aligned}\right\} \quad (1)$$

Let $N = S + I + Q + R + A$, then $\dot{N}(t) = T - \mu N$.

Let $\Omega = \{(S, I, Q, R, A): S, I, Q, R, A \leq 0, S + I + Q + R + A < T/\mu\}$ (2)

Here consider that the influx rate $T = 0 \Rightarrow \mu = 0$. In fact everyday new computers can be incorporated to the network or old computers can be removed from the network. In this paper we analyse the dynamical behavior of the spread of the virus in adding new computers or removing the old computers.

3. The virus free equilibrium point

When $I = 0$,

If $A = 0$, the viruses free equilibrium of system (1) is

$P_1 = (S_1, I_1, Q_1, R_1, A_1) = (T/\mu, 0,0,0,0)$

If $A \neq 0$, the virus free equilibrium of system (1) is

$$P_2 = (S_2, I_2, Q_2, R_2, A_2) = \left(\frac{\mu}{\alpha_{SA}}, 0,0,0, \frac{T}{\mu} - \frac{\mu}{\alpha_{SA}}\right)$$

We obtain $R_{01} = T\alpha_{SA}/\mu$

4. The Endemic equilibrium point

Assuming that infection exist in the network i.e.; $I \neq 0 \Rightarrow R \neq 0$ and considering $A = 0$, then there exists an endemic equilibrium point $P_3 = (S_3, I_3, Q_3, R_3, A_3)$.

When $A = 0$ using second equilibrium of system (1), we have

$$S = \frac{\mu + \delta}{\beta}$$

From third equation of the system (1), we have

$$Q = \frac{\delta I}{\mu + \gamma}$$

From fourth equation of the system (1), we have

$$R = \frac{\gamma\delta I}{(\mu + \rho)(\mu + \gamma)}$$

From the first equation of the system (1), we obtain

$$I = \frac{\{T\beta - \mu(\mu + \delta)\}(\mu + \rho)(\mu + \gamma)}{\beta\mu\{(\mu + \rho)(\mu + \gamma + \delta) + \gamma\delta\}}$$

Therefore, when the basic reproduction number R_{02} is calculated is

$$R_{02} = \frac{T\beta}{\mu(\mu + \delta)}$$

When $R_{02} > 1$, we have the positive equilibrium point

$$P_3 = (S_3, I_3, Q_3, R_3, A_3) = \left(\frac{\mu+\delta}{\beta}, \frac{\{T\beta - \mu(\mu+\delta)\}(\mu+\rho)(\mu+\gamma)}{\beta\mu\{(\mu+\rho)(\mu+\gamma+\delta)+\gamma\delta\}}, \frac{\delta I_3}{\mu+\gamma}, \frac{\gamma\delta I_3}{(\mu+\rho)(\mu+\gamma)}, 0\right)$$

$$= \left(\frac{\mu+\delta}{\beta}, \frac{(R_{02}-1)(\mu+\delta)(\mu+\rho)(\mu+\gamma)}{\beta\{(\mu+\rho)(\mu+\gamma+\delta)+\gamma\delta\}}, \frac{\delta I_3}{\mu+\gamma}, \frac{\gamma\delta I_3}{(\mu+\rho)(\mu+\gamma)}, 0\right)$$

5. Stability of virus free equilibrium points P_1

$$J_{P_1} = \begin{bmatrix} -\mu & -\beta T/\mu & 0 & \rho & -(\alpha_{SA}T)/\mu \\ 0 & \beta T/\mu - \mu - \delta & 0 & 0 & 0 \\ 0 & \delta & -(\mu+\gamma) & 0 & 0 \\ 0 & 0 & \gamma & -(\mu+\rho) & 0 \\ 0 & 0 & 0 & 0 & (\alpha_{SA}T)/\mu - \mu \end{bmatrix}$$

The characteristic equation of J_{P_1} is given by $|J_{P_1} - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} -\mu-\lambda & -\beta T/\mu & 0 & \rho & -(\alpha_{SA}T)/\mu \\ 0 & \beta T/\mu - \mu - \delta - \lambda & 0 & 0 & 0 \\ 0 & \delta & -(\mu+\gamma+\lambda) & 0 & 0 \\ 0 & 0 & \gamma & -(\mu+\rho+\lambda) & 0 \\ 0 & 0 & 0 & 0 & (\alpha_{SA}T)/\mu - \mu - \lambda \end{vmatrix} = 0$$

$$\Rightarrow (\mu+\gamma)\left(\frac{\beta T}{\mu} - \mu - \delta - \lambda\right)(\mu+\gamma+\lambda)(\mu+\rho+\lambda)\left(\mu - \frac{\alpha_{SA}T}{\mu} + \lambda\right) = 0$$

Here $\frac{\beta T}{\mu} - \mu - \delta < 0$ iff $R_{02} < 1$ and $-\frac{T\alpha_{SA}}{\mu} + \mu > 0$ iff $R_{01} < 1$.

it follows from the Routh-Hurwitz criterion that the eigen values have negative real parts, iff $R_{01} < 1$ and $R_{02} < 1$. Hence the virus free equilibrium point P_1 of the system (1) is locally asymptotically stable.

If $R_{01} > 1$, then the virus free equilibrium point P_1 is unstable and system (1) exhibits to other virus free equilibrium point P_2 $(S_2, I_2, Q_2, R_2, A_2)$.

If $R_{02} > 1$, then the virus free equilibrium point P_2 is unstable and system (1) exhibits to other virus free equilibrium point P_3 $(S_3, I_3, Q_3, R_3, A_3)$.

$$J_{P_2} = \begin{bmatrix} -\alpha_{SA}A_2 - \mu & -\beta S_2 & 0 & \rho & -\alpha_{SA}S_2 \\ 0 & \beta S_2 - \alpha_{IA}A_2 - \mu - \delta & 0 & 0 & 0 \\ 0 & \delta & -(\mu+\gamma) & 0 & 0 \\ 0 & 0 & \gamma & -(\mu+\rho) & 0 \\ \alpha_{SA}A_2 & \alpha_{IA}A_2 & 0 & 0 & 0 \end{bmatrix}$$

where $A_2 = \frac{T}{\mu} - \frac{\mu}{\alpha_{SA}}$ and $S_2 = \frac{\mu}{\alpha_{SA}}$

The characteristic equation of J_{P_2} is given by $|J_{P_2} - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} -\alpha_{SA}A_2 - \mu - \lambda & -\beta S_2 & 0 & \rho & -\alpha_{SA}S_2 \\ 0 & \beta S_2 - \alpha_{IA}A_2 - \mu - \delta - \lambda & 0 & 0 & 0 \\ 0 & \delta & -(\mu+\gamma+\lambda) & 0 & 0 \\ 0 & 0 & \gamma & -(\mu+\rho+\lambda) & 0 \\ \alpha_{SA}A_2 & \alpha_{IA}A_2 & 0 & 0 & -\lambda \end{vmatrix} = 0$$

$$\Rightarrow (\alpha_{SA}A_2 + \mu + \lambda)(\beta S_2 - \alpha_{IA}A_2 - \mu - \delta - \lambda)(\mu+\gamma+\lambda)(\mu+\rho+\lambda)\lambda$$
$$+ \alpha_{SA}A_2(\alpha_{SA}S_2)(\mu+\rho+\lambda)(\beta S_2 - \alpha_{IA}A_2 - \mu - \delta - \lambda)(\mu+\gamma+\lambda) = 0$$

$$\Rightarrow (\beta S_2 - \alpha_{IA}A_2 - \mu - \delta - \lambda)(\mu+\gamma+\lambda)(\mu+\rho+\lambda)\{(\alpha_{SA}A_2 + \mu + \lambda)\lambda + {\alpha_{SA}}^2 A_2 S_2\} = 0$$

We obtain five characteristic root i.e.,

$\beta S_2 - \alpha_{IA}A_2 - \mu - \delta$, $-(\mu+\rho)$, $-(\mu+\gamma)$, $-\mu$ and $\mu - \frac{T\alpha_{SA}}{\mu} \equiv -\alpha_{SA}A_2$

When $R_{01} > 1$ then $\beta S_2 - \alpha_{IA}A_2 - \mu - \delta < 0$ iff $R_{03} = \frac{(\beta + \alpha_{IA})\mu^2}{T\alpha_{IA} + \mu(\mu+\delta)\alpha_{SA}} < 1$

Hence the virus free equilibrium point P_2 of the system (1) is locally stable. i.e. if $R_{01} > 1$ and $R_{03} < 1$.

Now if $R_{01} > 1$ and $R_{03} > 1$ the virus free equilibrium point P_2 of the system (1) is unstable.

Theorem : (a) If $R_{01} < 1$ and $R_{02} < 1$, the virus free equilibrium point P_1 of the system (1) is stable
(b) If $R_{01} > 1$, then the virus free equilibrium point P_1 of the system (1) is unstable and system (1) exhibits to other virus free equilibrium point P_2 .
(c) If $R_{02} > 1$, then the virus free equilibrium point P_2 of the system (1) is unstable and system (1) exhibits to endemic equilibrium point P_3.
(d) If $R_{01} > 1$ and $R_{03} < 1$ the virus free equilibrium point P_2 is stable.
(e) If $R_{01} > 1$ and $R_{03} > 1$ the virus free equilibrium point P_2 is unstable.

6. Stability of the Endemic equilibrium points P_3

$$J_{P_3} = \begin{bmatrix} -\mu-\beta I_3 & -\beta S_3 & 0 & \rho & -\alpha_{SA}S_3 \\ \beta I_3 & \beta S_3-\mu-\delta & 0 & 0 & -\alpha_{IA}I_3 \\ 0 & \delta & -(\mu+\gamma) & 0 & 0 \\ 0 & 0 & \gamma & -(\mu+\rho) & 0 \\ 0 & 0 & 0 & 0 & \alpha_{SA}S_3+\alpha_{IA}I_3-\mu \end{bmatrix}$$

The characteristic equation of J_{P_3} is given by $|J_{P_3} - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} -\mu-\beta I_3-\lambda & -\beta S_3 & 0 & \rho & -\alpha_{SA}S_3 \\ \beta I_3 & \beta S_3-\mu-\delta-\lambda & 0 & 0 & -\alpha_{IA}I_3 \\ 0 & \delta & -(\mu+\gamma+\lambda) & 0 & 0 \\ 0 & 0 & \gamma & -(\mu+\rho+\lambda) & 0 \\ 0 & 0 & 0 & 0 & \alpha_{SA}S_3+\alpha_{IA}I_3-\mu-\lambda \end{vmatrix} = 0$$

$\Rightarrow (\alpha_{SA}S_3 + \alpha_{IA}I_3 - \mu - \lambda)\, h(\lambda) = 0$ where $h(\lambda) = \lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4$

$a_1 = 4\mu + \gamma + \rho + \delta + \beta I_3 - \beta S_3$

$a_2 = 3\mu(2\mu + \rho + \gamma + \delta - \beta S_3 + \beta I_3) + \beta(\delta + \rho + \gamma)I_3 - \beta(\gamma + \rho)S_3 + \rho\delta + \delta\gamma + \gamma\rho$

$a_3 = (2\mu + \rho + \gamma)(\mu^2 + \mu\delta - \beta\mu S_3 + \beta\mu I_3 + \beta\delta I_3) + (2\mu + \delta + \beta I_3 - \beta S_3)(\mu + \rho)(\mu + \gamma)$

$a_4 = (\mu + \rho)(\mu + \gamma)(\mu^2 + \mu\delta - \beta\mu S_3 + \beta\mu I_3 + \beta\delta I_3)$

By Routh-Hurwitz Criterion, Here $a_1a_2a_3 - {a_3}^2 - {a_1}^2a_4 > 0$

Hence, the eigenvalues of $h(\lambda) = 0$, have negative real parts.

Therefore the eigenvalues of the characteristic equation in J_{P_3} have negative real parts, only

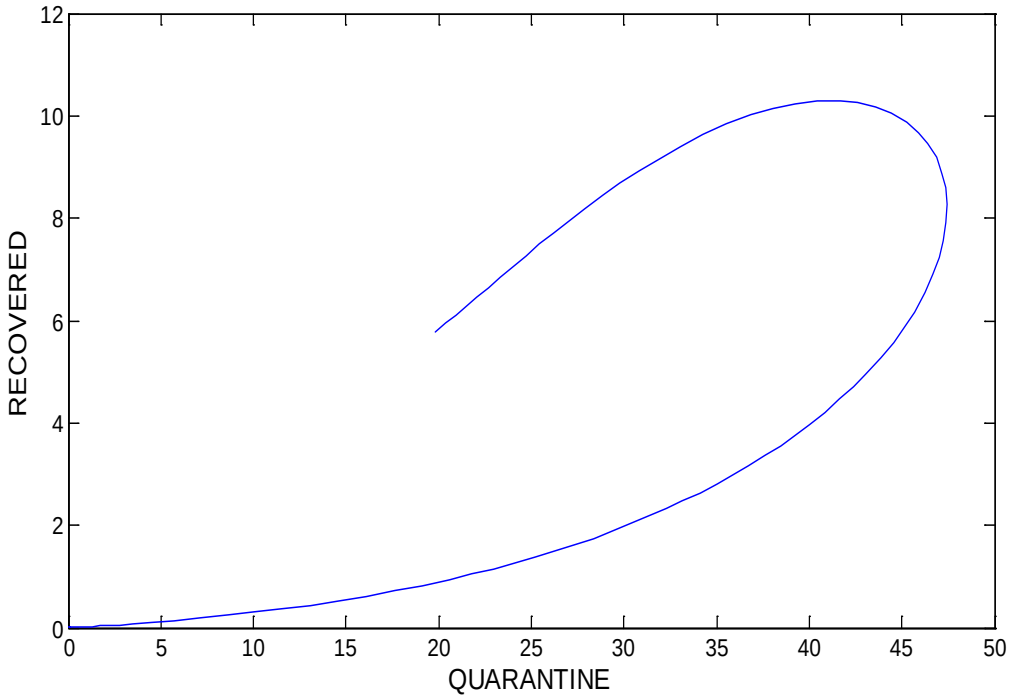
$$\alpha_{SA}S_3 + \alpha_{IA}I_3 - \mu < 0 \qquad \Rightarrow \alpha_{SA}\left(\frac{\mu+\delta}{\beta}\right) + \alpha_{IA}I_3 - \mu < 0$$

$$\Rightarrow \alpha_{IA}I_3 < \mu - \alpha_{SA}\left(\frac{\mu+\delta}{\beta}\right)$$

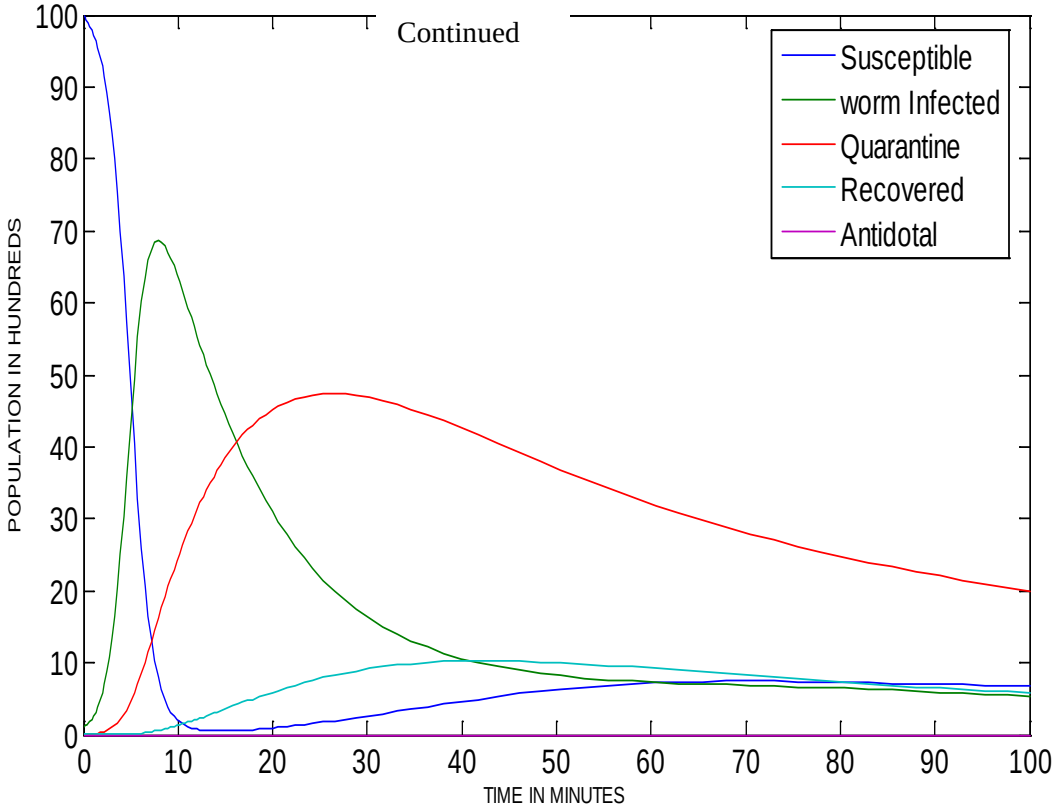
Therefore, $$R_{04} = \frac{\alpha_{IA}I_3}{\mu - \alpha_{SA}\left(\frac{\mu+\delta}{\beta}\right)} < 1$$

7. Conclusion

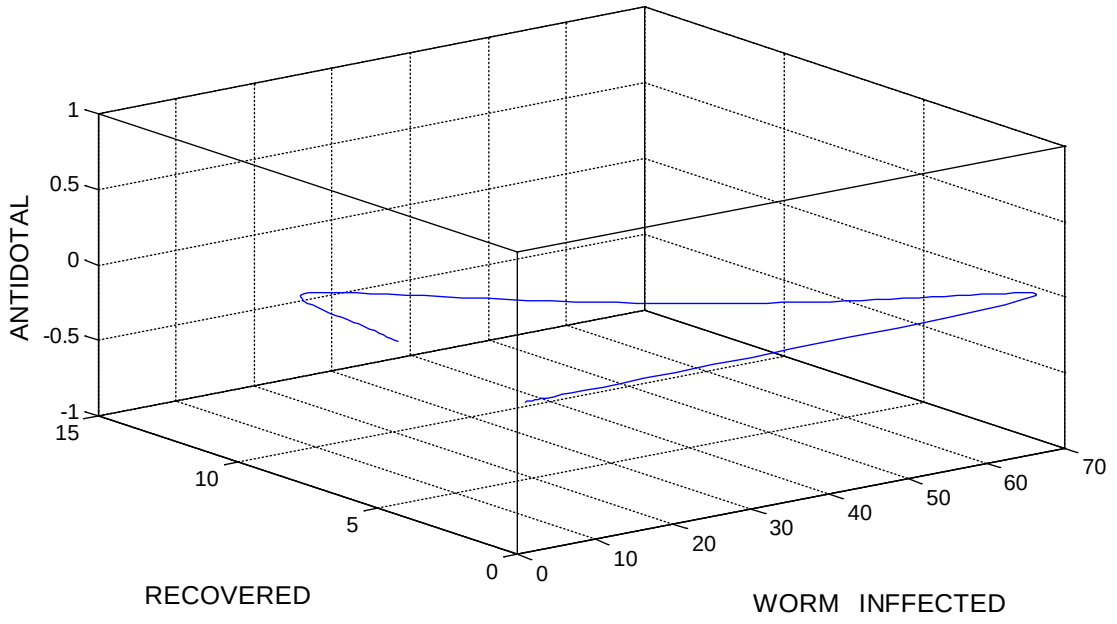
In this article, the proposed model is SIQRA. This system is an extention of SAIR model.This model is developed for getting Virus Free Equilibrium (VFE) point. We discussed about stability of VFE point and Endemic Equilibrium (EE) point. By Routh-Hurwitz criterion we discussed the reproduction number R_{o1}in VFE and R_{o2} EE point. At VFE point if $R_{o1} < 1$ then VFE point is stable and if $R_{o1} > 1$ then VFE point is unstable. In 2-dimenssion and 3- dimension several graphs of parameters are discussed. The analysis can control the virus in all computer network

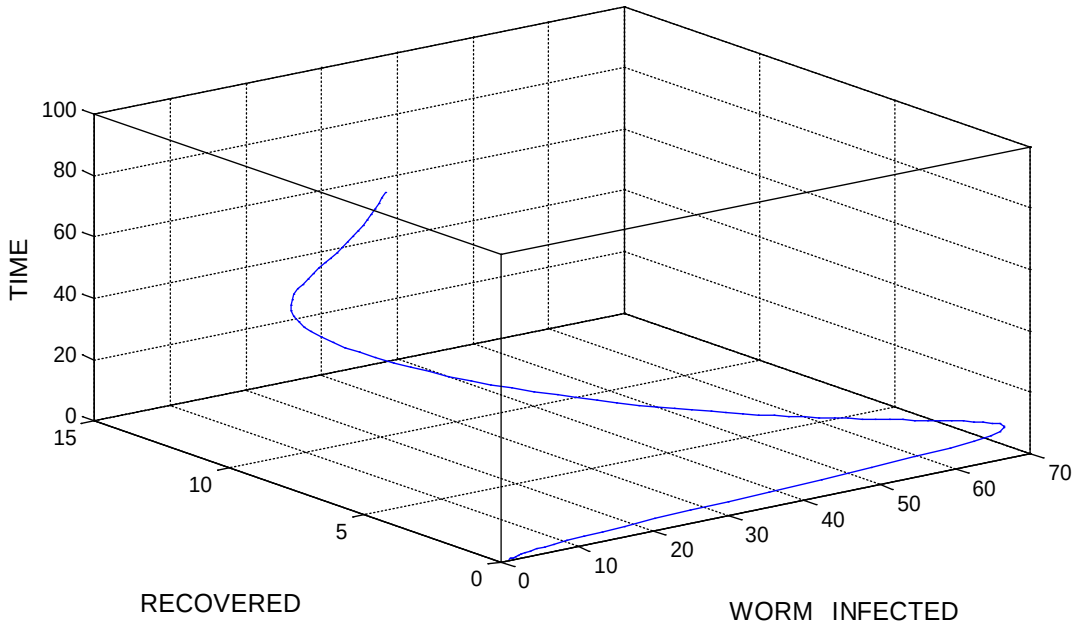


Quarantine nodes verses Recovered nodes

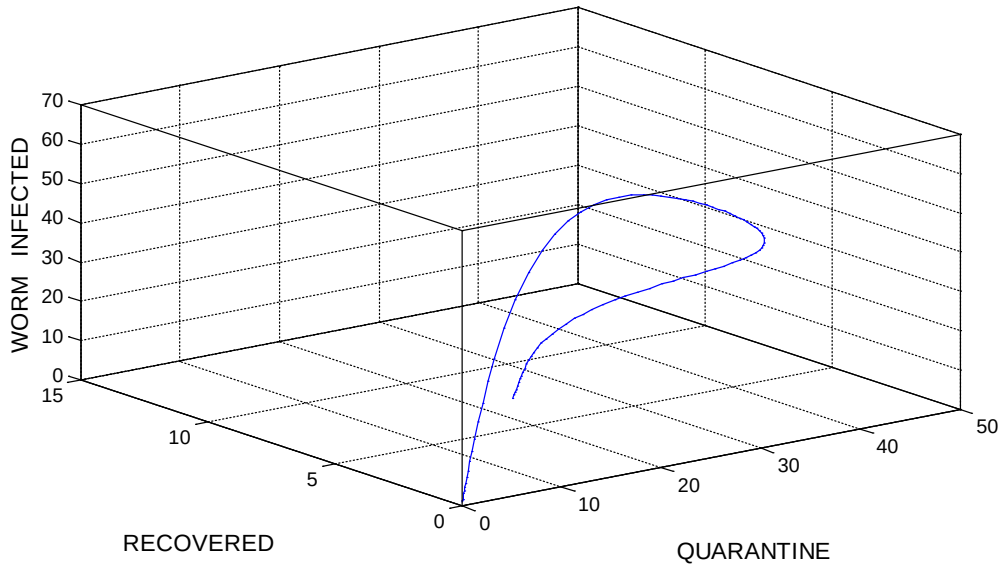


Dynamic behavior of the system (1)

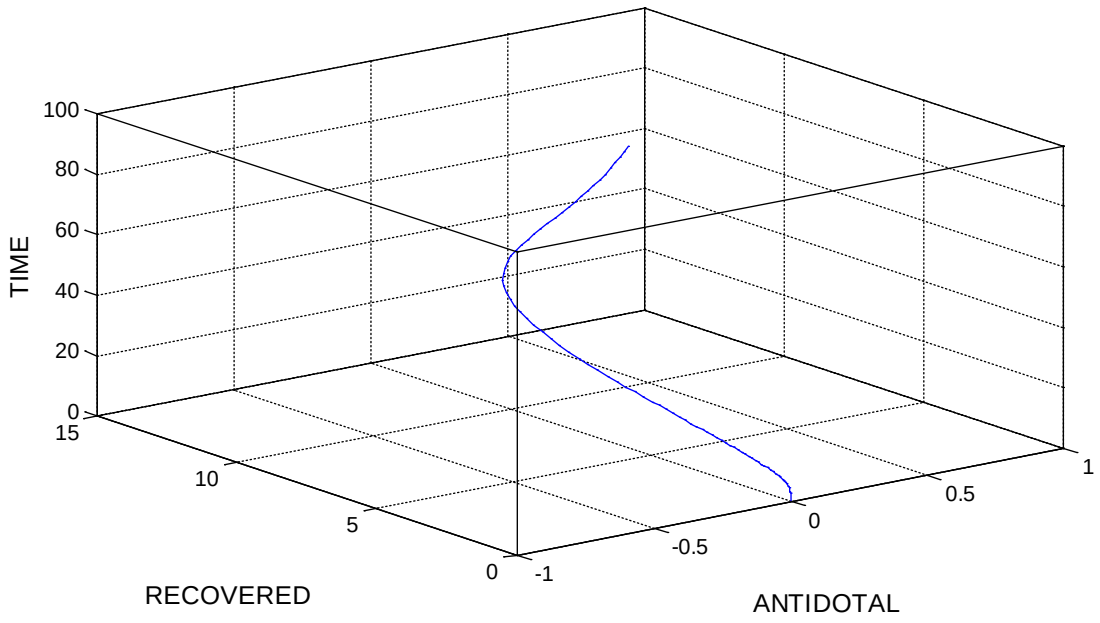




Worm Infected nodes verses Recovered nodes



Quarantine nodes verses Recovered nodes



Antidotal nodes verses Recovered nodes

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